Math 55: Discrete Mathematics, Fall 2008
Homework 8 Solutions

*[3,3 and 4 points] 5.5: 10(a) $\binom{12+6-1}{12}$ (choosing 12 things from 6 with repetitions)

(c) $\binom{12+6-1}{12}$ (the first two of each kind make 12 and you have a dozen left to choose freely)

(f) $\binom{15+6-1}{15} - \binom{11+6-1}{11}$ (count choices without any restriction on broccoli, then subtract those with at least 4 broccoli)

34. $\binom{5}{3,2} + \binom{5}{3,1,1} + \binom{5}{2,3} + \binom{5}{2,2,1} + \binom{5}{1,3,1} + \binom{6}{3,3} + \binom{6}{3,2,1} + \binom{6}{2,3,1} + \binom{7}{3,3,1}$ (one term for each possible number of E's, S's and R's adding up to at least 5)

42. $\binom{52}{13,13,13,13} = 52!/(13!)^4$

6.1: 8. $\binom{51}{4}/\binom{52}{5}$

16. $4\binom{13}{5}/\binom{52}{5}$

18. $4 \cdot 10/\binom{52}{5}$

30. $6 \cdot 34/\binom{40}{6}$ (I assume that the problem intends an *unordered* subset of 6 integers chosen at random from $\{1, \ldots, 40\}$. The numerator counts subsets that match exactly 5 of the player's chosen 6.)

36. 8 is more likely with two dice than three. With two dice, $p(8) = 5/36$. With three dice, $p(8) = 21/6^3$.

6.2: 8(a) 1/2 (half the permutations have 1 preceding 2 and the other half have 2 preceding 1)

(c) $1/n$ (compute it as $(n-1)!/n!$)

(d) 1/4 (intersection of two independent events with $p = 1/2$ each)

* 16. We must show $p(\overline{E} \cap \overline{F}) = p(\overline{E})p(\overline{F})$. The left hand side is equal to $1 - p(E \cup F) = 1 - \big(p(E) + p(F) - p(E \cap F)\big) = 1 - p(E) - p(F) + p(E)p(F)$, since E and F are independent. This factors as $(1 - p(E))(1 - p(F))$, which is equal to the right hand side of the desired identity.

34. (a) $p(s = 0) = (1-p)^n$

(b) $p(s \geq 1) = 1 - (1-p)^n$

(c) $p(s \leq 1) = (1-p)^n + np(1-p)^{n-1}$

(d) $p(s \geq 2) = 1 - p(s \leq 1) = 1 - \big((1-p)^n + np(1-p)^{n-1}\big)$

38. (a) 2/11

(b) 2/11 (same as the previous answer because either way there are 11 outcomes in the conditioning event, and two of them make 7).

* (A) We have

$$p(\text{four aces}|\text{red ace}) = p(\text{four aces})/p(\text{red ace}),$$

$$p(\text{four aces}|\text{heart ace}) = p(\text{four aces})/p(\text{heart ace}).$$

Since $p(\text{red ace}) > p(\text{heart ace})$ it follows that $p(\text{four aces}|\text{heart ace}) > p(\text{four aces}|\text{red ace})$. It is also possible, though not necessary, to calculate these probabilities exactly.

One way to better understand this somewhat counter-intuitive result is that the more specific information that there is a heart ace narrows the sample space more than the information that there is a red ace does.

(B) 40/100=2/5 (The integers with a multiplicative inverse are those relatively prime to 100, that is, they are odd and not divisible by 5. There are 40 of these among the 100 integers zero to 99.)