Math 55: Discrete Mathematics, Fall 2008
Homework 8 Solutions

* [3,3 and 4 points] 5.5: 10(a) \( \binom{12+6-1}{12} \) (choosing 12 things from 6 with repetitions)
(5) \( \binom{15+6-1}{15} - \binom{11+6-1}{11} \) (count choices without any restriction on broccoli, then subtract those with at least 4 broccoli)

34. \( \binom{5}{3,2} + \binom{5}{3,1,1} + \binom{5}{2,3} + \binom{5}{2,2,1} + \binom{5}{1,3,1} + \binom{6}{3,3} + \binom{6}{3,2,1} + \binom{6}{2,3,1} + \binom{7}{3,3,1} \) (one term for each possible number of E’s, S’s and R’s adding up to at least 5)

42. \( \binom{52}{13,13,13,13} = \frac{52!}{(13!)^4} \)

6.1: 8. \( \binom{51}{4} / \binom{52}{5} \)
16. \( 4\binom{13}{5} / \binom{52}{5} \)
18. \( 4 \cdot 10 / \binom{52}{5} \)

30. \( 6 \cdot 34 / \binom{40}{6} \) (I assume that the problem intends an unordered subset of 6 integers chosen at random from \( \{1, \ldots, 40\} \). The numerator counts subsets that match exactly 5 of the player’s chosen 6.)

36. 8 is more likely with two dice than three. With two dice, \( p(8) = \frac{5}{36} \). With three dice, \( p(8) = \frac{1}{216} \).

6.2: 8(a) 1/2 (half the permutations have 1 preceding 2 and the other half have 2 preceding 1)
(6) \( 1/n \) (compute it as \((n-1)!/n!\))
(d) \( 1/4 \) (intersection of two independent events with \( p = 1/2 \) each)

* 16. We must show \( p(E \cap F) = p(E) p(F) \). The left hand side is equal to \( 1 - p(E \cup F) = 1 - (p(E) + p(F) - p(E \cap F)) \) = \( 1 - p(E) - p(F) + p(E)p(F) \), since \( E \) and \( F \) are independent. This factors as \( (1 - p(E))(1 - p(F)) \), which is equal to the right hand side of the desired identity.

34. (a) \( p(s = 0) = (1 - p)^n \)
(b) \( p(s \geq 1) = 1 - (1 - p)^n \)
(c) \( p(s \leq 1) = (1 - p)^n + np(1 - p)^{n-1} \)
(d) \( p(s \geq 2) = 1 - p(s \leq 1) = 1 - ((1 - p)^n + np(1 - p)^{n-1}) \)

38. (a) 2/11
(b) 2/11 (same as the previous answer because either way there are 11 outcomes in the conditioning event, and two of them make 7).

* (A) We have
\[ p(\text{four aces}|\text{red ace}) = p(\text{four aces}) / p(\text{red ace}), \]
\[ p(\text{four aces}|\text{heart ace}) = p(\text{four aces}) / p(\text{heart ace}). \]

Since \( p(\text{red ace}) > p(\text{heart ace}) \) it follows that \( p(\text{four aces}|\text{heart ace}) > p(\text{four aces}|\text{red ace}) \). It is also possible, though not necessary, to calculate these probabilities exactly.
One way to better understand this somewhat counter-intuitive result is that the more specific information that there is a heart ace narrows the sample space more than the information that there is a red ace does.

(B) $\frac{40}{100} = \frac{2}{5}$ (The integers with a multiplicative inverse are those relatively prime to 100, that is, they are odd and not divisible by 5. There are 40 of these among the 100 integers zero to 99.)