

# Generating function for connected graphs

Calculating  $Z_{\text{conn}}(x, q) = \sum_{n,k} g_{\text{conn}}(n, k) x^n q^k$ , where  $g_{\text{conn}}(n, k)$  is the number of isomorphism classes of connected simple graphs with  $n$  vertices and  $k$  edges.

```
<< Combinatorica`
```

General::compat :

Combinatorica Graph and Permutations functionality has been superseded by preloaded functionality. The package now being loaded may conflict with this. Please see the Compatibility Guide for details.

```
gp[lam_] :=
Product[(1 + qLCM[lam[[i]], lam[[j]]])GCD[lam[[i]], lam[[j]]], {i, Length[lam]}, {j, i - 1}]
Product[If[EvenQ[l], (1 + q1)1/2-1 (1 + q1/2), (1 + q1)(1-1)/2], {l, lam}]
```

```
z[lam_] := (Times @@ lam) Product[lr[[2]]!, {lr, Tally[lam]]}
```

Terms up to  $x^n$  in the generating function for isomorphism classes of possibly disconnected graphs.

```
zg[n_] := Collect[Sum[gp[lam] xk / z[lam], {k, n}, {lam, Partitions[k]}], x, Expand]
```

Take `plethystic logarithm` to get the generating function for connected graphs.

```
zconn[n_] := Collect[Normal[
Sum[MoebiusMu[k] Log[1 + (zg[n] /. {x -> xk, q -> qk}) + O[x]n+1] / k, {k, n}]], x, Expand]
```

**zconn [10]**

$$\begin{aligned}
& x + q x^2 + (q^2 + q^3) x^3 + (2 q^3 + 2 q^4 + q^5 + q^6) x^4 + \\
& (3 q^4 + 5 q^5 + 5 q^6 + 4 q^7 + 2 q^8 + q^9 + q^{10}) x^5 + \\
& (6 q^5 + 13 q^6 + 19 q^7 + 22 q^8 + 20 q^9 + 14 q^{10} + 9 q^{11} + 5 q^{12} + 2 q^{13} + q^{14} + q^{15}) x^6 + \\
& (11 q^6 + 33 q^7 + 67 q^8 + 107 q^9 + 132 q^{10} + 138 q^{11} + 126 q^{12} + \\
& \quad 95 q^{13} + 64 q^{14} + 40 q^{15} + 21 q^{16} + 10 q^{17} + 5 q^{18} + 2 q^{19} + q^{20} + q^{21}) x^7 + \\
& (23 q^7 + 89 q^8 + 236 q^9 + 486 q^{10} + 814 q^{11} + 1169 q^{12} + 1454 q^{13} + 1579 q^{14} + \\
& \quad 1515 q^{15} + 1290 q^{16} + 970 q^{17} + 658 q^{18} + 400 q^{19} + 220 q^{20} + \\
& \quad 114 q^{21} + 56 q^{22} + 24 q^{23} + 11 q^{24} + 5 q^{25} + 2 q^{26} + q^{27} + q^{28}) x^8 + \\
& (47 q^8 + 240 q^9 + 797 q^{10} + 2075 q^{11} + 4495 q^{12} + 8404 q^{13} + 13855 q^{14} + 20303 q^{15} + \\
& \quad 26631 q^{16} + 31400 q^{17} + 33366 q^{18} + 31996 q^{19} + 27764 q^{20} + 21817 q^{21} + \\
& \quad 15558 q^{22} + 10096 q^{23} + 5984 q^{24} + 3247 q^{25} + 1635 q^{26} + 770 q^{27} + 344 q^{28} + \\
& \quad 148 q^{29} + 63 q^{30} + 25 q^{31} + 11 q^{32} + 5 q^{33} + 2 q^{34} + q^{35} + q^{36}) x^9 + \\
& (106 q^9 + 657 q^{10} + 2678 q^{11} + 8548 q^{12} + 22950 q^{13} + 53863 q^{14} + 112618 q^{15} + 211866 q^{16} + \\
& \quad 361342 q^{17} + 561106 q^{18} + 795630 q^{19} + 1032754 q^{20} + 1229228 q^{21} + 1343120 q^{22} + \\
& \quad 1348674 q^{23} + 1245369 q^{24} + 1057896 q^{25} + 827086 q^{26} + 595418 q^{27} + 394820 q^{28} + \\
& \quad 241428 q^{29} + 136370 q^{30} + 71293 q^{31} + 34652 q^{32} + 15767 q^{33} + 6757 q^{34} + 2768 q^{35} + \\
& \quad 1102 q^{36} + 428 q^{37} + 165 q^{38} + 66 q^{39} + 26 q^{40} + 11 q^{41} + 5 q^{42} + 2 q^{43} + q^{44} + q^{45}) x^{10}
\end{aligned}$$

**zconn [20]**

$$\begin{aligned}
& x + q x^2 + (q^2 + q^3) x^3 + (2 q^3 + 2 q^4 + q^5 + q^6) x^4 + \\
& (3 q^4 + 5 q^5 + 5 q^6 + 4 q^7 + 2 q^8 + q^9 + q^{10}) x^5 + \\
& (6 q^5 + 13 q^6 + 19 q^7 + 22 q^8 + 20 q^9 + 14 q^{10} + 9 q^{11} + 5 q^{12} + 2 q^{13} + q^{14} + q^{15}) x^6 + \\
& (11 q^6 + 33 q^7 + 67 q^8 + 107 q^9 + 132 q^{10} + 138 q^{11} + 126 q^{12} + \\
& \quad 95 q^{13} + 64 q^{14} + 40 q^{15} + 21 q^{16} + 10 q^{17} + 5 q^{18} + 2 q^{19} + q^{20} + q^{21}) x^7 + \\
& (23 q^7 + 89 q^8 + 236 q^9 + 486 q^{10} + 814 q^{11} + 1169 q^{12} + 1454 q^{13} + 1579 q^{14} + \\
& \quad 1515 q^{15} + 1290 q^{16} + 970 q^{17} + 658 q^{18} + 400 q^{19} + 220 q^{20} + \\
& \quad 114 q^{21} + 56 q^{22} + 24 q^{23} + 11 q^{24} + 5 q^{25} + 2 q^{26} + q^{27} + q^{28}) x^8 + \\
& (47 q^8 + 240 q^9 + 797 q^{10} + 2075 q^{11} + 4495 q^{12} + 8404 q^{13} + 13855 q^{14} + 20303 q^{15} + \\
& \quad 26631 q^{16} + 31400 q^{17} + 33366 q^{18} + 31996 q^{19} + 27764 q^{20} + 21817 q^{21} + \\
& \quad 15558 q^{22} + 10096 q^{23} + 5984 q^{24} + 3247 q^{25} + 1635 q^{26} + 770 q^{27} + 344 q^{28} + \\
& \quad 148 q^{29} + 63 q^{30} + 25 q^{31} + 11 q^{32} + 5 q^{33} + 2 q^{34} + q^{35} + q^{36}) x^9 + \\
& (106 q^9 + 657 q^{10} + 2678 q^{11} + 8548 q^{12} + 22950 q^{13} + 53863 q^{14} + 112618 q^{15} + 211866 q^{16} + \\
& \quad 361342 q^{17} + 561106 q^{18} + 795630 q^{19} + 1032754 q^{20} + 1229228 q^{21} + 1343120 q^{22} + \\
& \quad 1348674 q^{23} + 1245369 q^{24} + 1057896 q^{25} + 827086 q^{26} + 595418 q^{27} + 394820 q^{28} + \\
& \quad 241428 q^{29} + 136370 q^{30} + 71293 q^{31} + 34652 q^{32} + 15767 q^{33} + 6757 q^{34} + 2768 q^{35} + \\
& \quad 1102 q^{36} + 428 q^{37} + 165 q^{38} + 66 q^{39} + 26 q^{40} + 11 q^{41} + 5 q^{42} + 2 q^{43} + q^{44} + q^{45}) x^{10} + \\
& (235 q^{10} + 1806 q^{11} + 8833 q^{12} + 33851 q^{13} + 109844 q^{14} + 313670 q^{15} + 803905 q^{16} + \\
& \quad 1870168 q^{17} + 3978187 q^{18} + 7775398 q^{19} + 14013042 q^{20} + 23350556 q^{21} + 36052412 q^{22} + \\
& \quad 51662585 q^{23} + 68803769 q^{24} + 85251441 q^{25} + 98355794 q^{26} + 105723785 q^{27} + \\
& \quad 105925685 q^{28} + 98945882 q^{29} + 86182087 q^{30} + 69994055 q^{31} + 53002668 q^{32} + \\
& \quad 37417977 q^{33} + 24624123 q^{34} + 15105276 q^{35} + 8639030 q^{36} + 4608750 q^{37} + 2295733 q^{38} +
\end{aligned}$$

$$\begin{aligned}
 & 1\,069\,824\,q^{39} + 467\,781\,q^{40} + 192\,777\,q^{41} + 75\,410\,q^{42} + 28\,257\,q^{43} + 10\,249\,q^{44} + 3663\,q^{45} + \\
 & 1305\,q^{46} + 467\,q^{47} + 172\,q^{48} + 67\,q^{49} + 26\,q^{50} + 11\,q^{51} + 5\,q^{52} + 2\,q^{53} + q^{54} + q^{55} \Big) x^{11} + \\
 & (551\,q^{11} + 5026\,q^{12} + 28\,908\,q^{13} + 130\,365\,q^{14} + 499\,888\,q^{15} + 1\,694\,642\,q^{16} + 5\,184\,582\,q^{17} + \\
 & 14\,484\,609\,q^{18} + 37\,234\,106\,q^{19} + 88\,525\,650\,q^{20} + 195\,410\,521\,q^{21} + 401\,665\,379\,q^{22} + 770\,641\,945\,q^{23} + \\
 & 1\,382\,812\,173\,q^{24} + 2\,324\,370\,686\,q^{25} + 3\,665\,000\,607\,q^{26} + 5\,427\,096\,867\,q^{27} + 7\,554\,423\,219\,q^{28} + \\
 & 9\,892\,755\,100\,q^{29} + 12\,195\,279\,971\,q^{30} + 14\,159\,301\,249\,q^{31} + 15\,489\,241\,571\,q^{32} + 15\,968\,704\,512\,q^{33} + \\
 & 15\,517\,703\,745\,q^{34} + 14\,214\,388\,740\,q^{35} + 12\,273\,198\,300\,q^{36} + 9\,987\,729\,903\,q^{37} + 7\,659\,048\,949\,q^{38} + \\
 & 5\,533\,185\,036\,q^{39} + 3\,764\,795\,097\,q^{40} + 2\,411\,768\,702\,q^{41} + 1\,454\,190\,308\,q^{42} + 825\,047\,242\,q^{43} + \\
 & 440\,383\,354\,q^{44} + 221\,143\,686\,q^{45} + 104\,503\,035\,q^{46} + 46\,505\,142\,q^{47} + 19\,515\,189\,q^{48} + 7\,739\,534\,q^{49} + \\
 & 2\,911\,060\,q^{50} + 1\,043\,763\,q^{51} + 359\,302\,q^{52} + 119\,888\,q^{53} + 39\,242\,q^{54} + 12\,762\,q^{55} + 4\,191\,q^{56} + \\
 & 1\,405\,q^{57} + 485\,q^{58} + 175\,q^{59} + 68\,q^{60} + 26\,q^{61} + 11\,q^{62} + 5\,q^{63} + 2\,q^{64} + q^{65} + q^{66} \Big) x^{12} + \\
 & (1301\,q^{12} + 13\,999\,q^{13} + 93\,569\,q^{14} + 489\,387\,q^{15} + 2\,179\,949\,q^{16} + 8\,610\,789\,q^{17} + 30\,809\,058\,q^{18} + \\
 & 101\,093\,020\,q^{19} + 306\,619\,793\,q^{20} + 864\,317\,147\,q^{21} + 2\,273\,437\,235\,q^{22} + 5\,597\,433\,239\,q^{23} + \\
 & 12\,932\,769\,342\,q^{24} + 28\,100\,037\,365\,q^{25} + 57\,519\,070\,110\,q^{26} + 111\,089\,419\,502\,q^{27} + \\
 & 202\,705\,284\,331\,q^{28} + 349\,854\,951\,952\,q^{29} + 571\,708\,043\,354\,q^{30} + 885\,315\,952\,770\,q^{31} + \\
 & 1\,300\,116\,617\,536\,q^{32} + 1\,811\,764\,113\,815\,q^{33} + 2\,397\,114\,478\,044\,q^{34} + 3\,012\,567\,495\,251\,q^{35} + \\
 & 3\,597\,513\,119\,353\,q^{36} + 4\,083\,272\,448\,966\,q^{37} + 4\,406\,012\,122\,869\,q^{38} + 4\,520\,384\,306\,832\,q^{39} + \\
 & 4\,409\,911\,747\,398\,q^{40} + 4\,090\,861\,008\,231\,q^{41} + 3\,608\,355\,629\,886\,q^{42} + 3\,025\,996\,522\,703\,q^{43} + \\
 & 2\,412\,253\,931\,236\,q^{44} + 1\,827\,602\,341\,191\,q^{45} + 1\,315\,624\,835\,444\,q^{46} + 899\,585\,775\,383\,q^{47} + \\
 & 584\,070\,320\,028\,q^{48} + 359\,946\,928\,749\,q^{49} + 210\,471\,375\,962\,q^{50} + 116\,723\,367\,957\,q^{51} + \\
 & 61\,371\,935\,016\,q^{52} + 30\,583\,533\,061\,q^{53} + 14\,441\,431\,145\,q^{54} + 6\,461\,044\,052\,q^{55} + 2\,739\,256\,828\,q^{56} + \\
 & 1\,101\,081\,723\,q^{57} + 420\,048\,320\,q^{58} + 152\,339\,777\,q^{59} + 52\,664\,789\,q^{60} + 17\,422\,958\,q^{61} + \\
 & 5\,546\,608\,q^{62} + 1\,711\,903\,q^{63} + 517\,119\,q^{64} + 154\,657\,q^{65} + 46\,414\,q^{66} + 14\,140\,q^{67} + 4\,435\,q^{68} + \\
 & 1\,446\,q^{69} + 492\,q^{70} + 176\,q^{71} + 68\,q^{72} + 26\,q^{73} + 11\,q^{74} + 5\,q^{75} + 2\,q^{76} + q^{77} + q^{78} \Big) x^{13} + \\
 & (3159\,q^{13} + 39\,260\,q^{14} + 300\,748\,q^{15} + 1\,799\,700\,q^{16} + 9\,181\,185\,q^{17} + 41\,613\,518\,q^{18} + \\
 & 171\,301\,235\,q^{19} + 648\,723\,984\,q^{20} + 2\,278\,745\,633\,q^{21} + 7\,466\,604\,072\,q^{22} + 22\,916\,781\,627\,q^{23} + \\
 & 66\,099\,464\,855\,q^{24} + 179\,638\,437\,584\,q^{25} + 461\,011\,580\,013\,q^{26} + 1\,119\,306\,632\,640\,q^{27} + \\
 & 2\,575\,222\,889\,571\,q^{28} + 5\,622\,435\,575\,500\,q^{29} + 11\,663\,261\,941\,318\,q^{30} + 23\,013\,407\,529\,653\,q^{31} + \\
 & 43\,234\,705\,737\,618\,q^{32} + 77\,402\,043\,968\,500\,q^{33} + 132\,152\,655\,846\,345\,q^{34} + 215\,327\,701\,580\,998\,q^{35} + \\
 & 335\,032\,391\,116\,513\,q^{36} + 498\,043\,461\,420\,246\,q^{37} + 707\,689\,564\,672\,453\,q^{38} + 961\,585\,825\,489\,930\,q^{39} + \\
 & 1\,249\,834\,675\,602\,570\,q^{40} + 1\,554\,398\,715\,428\,131\,q^{41} + 1\,850\,212\,617\,950\,038\,q^{42} + \\
 & 2\,108\,207\,254\,742\,072\,q^{43} + 2\,299\,855\,868\,850\,890\,q^{44} + 2\,402\,302\,590\,759\,788\,q^{45} + \\
 & 2\,402\,814\,904\,220\,039\,q^{46} + 2\,301\,369\,651\,720\,480\,q^{47} + 2\,110\,651\,394\,921\,418\,q^{48} + \\
 & 1\,853\,464\,867\,161\,397\,q^{49} + 1\,558\,285\,275\,004\,448\,q^{50} + 1\,254\,137\,166\,462\,738\,q^{51} + 966\,058\,045\,338\,745\,q^{52} + \\
 & 712\,082\,670\,286\,631\,q^{53} + 502\,134\,606\,351\,257\,q^{54} + 338\,650\,016\,859\,694\,q^{55} + 218\,367\,392\,720\,691\,q^{56} + \\
 & 134\,580\,341\,919\,994\,q^{57} + 79\,245\,094\,691\,327\,q^{58} + 44\,564\,630\,072\,987\,q^{59} + 23\,925\,391\,528\,278\,q^{60} + \\
 & 12\,257\,444\,499\,522\,q^{61} + 5\,990\,150\,457\,761\,q^{62} + 2\,791\,314\,715\,482\,q^{63} + 1\,239\,858\,957\,825\,q^{64} + \\
 & 524\,838\,134\,114\,q^{65} + 211\,701\,682\,450\,q^{66} + 81\,379\,034\,600\,q^{67} + 29\,823\,510\,761\,q^{68} + 10\,428\,155\,017\,q^{69} + \\
 & 3\,483\,730\,631\,q^{70} + 1\,114\,245\,976\,q^{71} + 342\,260\,225\,q^{72} + 101\,403\,919\,q^{73} + 29\,148\,837\,q^{74} + \\
 & 8\,191\,602\,q^{75} + 2\,271\,858\,q^{76} + 628\,634\,q^{77} + 175\,651\,q^{78} + 50\,159\,q^{79} + 14\,775\,q^{80} + 4\,541\,q^{81} + \\
 & 1\,464\,q^{82} + 495\,q^{83} + 177\,q^{84} + 68\,q^{85} + 26\,q^{86} + 11\,q^{87} + 5\,q^{88} + 2\,q^{89} + q^{90} + q^{91} \Big) x^{14} + \\
 & (7741\,q^{14} + 110\,381\,q^{15} + 959\,374\,q^{16} + 6\,499\,706\,q^{17} + 37\,539\,466\,q^{18} + 192\,842\,712\,q^{19} + \\
 & 901\,377\,310\,q^{20} + 3\,885\,009\,655\,q^{21} + 15\,572\,583\,105\,q^{22} + 58\,393\,017\,190\,q^{23} + 205\,715\,634\,053\,q^{24} + \\
 & 683\,177\,510\,788\,q^{25} + 2\,144\,541\,235\,919\,q^{26} + 6\,377\,475\,240\,399\,q^{27} + 18\,001\,615\,191\,104\,q^{28} + \\
 & 48\,311\,050\,541\,875\,q^{29} + 123\,449\,595\,358\,013\,q^{30} + 300\,749\,354\,421\,503\,q^{31} + 699\,350\,108\,423\,184\,q^{32} + \\
 & 1\,553\,855\,733\,393\,286\,q^{33} + 3\,301\,861\,994\,911\,062\,q^{34} + 6\,715\,921\,394\,433\,155\,q^{35} + \\
 & 13\,085\,237\,079\,085\,229\,q^{36} + 24\,439\,136\,746\,521\,945\,q^{37} + 43\,781\,285\,251\,726\,544\,q^{38} +
 \end{aligned}$$

$$\begin{aligned}
& 75\,271\,977\,853\,717\,106\,q^{39} + 124\,262\,478\,451\,794\,760\,q^{40} + 197\,063\,431\,080\,433\,042\,q^{41} + \\
& 300\,336\,662\,510\,870\,107\,q^{42} + 440\,052\,795\,351\,963\,640\,q^{43} + 620\,062\,341\,104\,086\,439\,q^{44} + \\
& 840\,472\,214\,620\,329\,758\,q^{45} + 1\,096\,166\,036\,361\,399\,038\,q^{46} + 1\,375\,900\,185\,773\,096\,860\,q^{47} + \\
& 1\,662\,392\,479\,364\,026\,361\,q^{48} + 1\,933\,664\,512\,190\,616\,574\,q^{49} + 2\,165\,616\,006\,894\,383\,011\,q^{50} + \\
& 2\,335\,467\,291\,963\,665\,033\,q^{51} + 2\,425\,409\,960\,013\,204\,929\,q^{52} + 2\,425\,664\,021\,535\,713\,098\,q^{53} + \\
& 2\,336\,219\,576\,723\,209\,859\,q^{54} + 2\,166\,836\,191\,744\,515\,302\,q^{55} + 1\,935\,300\,544\,660\,143\,022\,q^{56} + \\
& 1\,664\,370\,030\,624\,532\,870\,q^{57} + 1\,378\,124\,524\,832\,488\,687\,q^{58} + 1\,098\,527\,430\,882\,329\,780\,q^{59} + \\
& 842\,854\,834\,413\,817\,653\,q^{60} + 622\,355\,382\,845\,277\,188\,q^{61} + 442\,161\,655\,252\,206\,509\,q^{62} + \\
& 302\,191\,796\,209\,764\,061\,q^{63} + 198\,625\,110\,119\,074\,266\,q^{64} + 125\,520\,745\,606\,786\,906\,q^{65} + \\
& 76\,242\,313\,958\,512\,824\,q^{66} + 44\,497\,427\,067\,016\,907\,q^{67} + 24\,944\,883\,481\,057\,701\,q^{68} + \\
& 13\,426\,927\,548\,992\,045\,q^{69} + 6\,936\,717\,090\,864\,854\,q^{70} + 3\,438\,284\,317\,260\,417\,q^{71} + \\
& 1\,634\,428\,043\,940\,840\,q^{72} + 744\,822\,913\,114\,198\,q^{73} + 325\,264\,654\,890\,834\,q^{74} + 136\,070\,581\,999\,699\,q^{75} + \\
& 54\,513\,625\,938\,028\,q^{76} + 20\,910\,535\,568\,729\,q^{77} + 7\,678\,976\,705\,817\,q^{78} + 2\,699\,941\,812\,194\,q^{79} + \\
& 909\,199\,464\,828\,q^{80} + 293\,429\,716\,395\,q^{81} + 90\,859\,877\,283\,q^{82} + 27\,040\,031\,520\,q^{83} + 7\,753\,406\,712\,q^{84} + \\
& 2\,149\,523\,514\,q^{85} + 578\,891\,690\,q^{86} + 152\,374\,454\,q^{87} + 39\,500\,164\,q^{88} + 10\,176\,658\,q^{89} + \\
& 2\,632\,419\,q^{90} + 691\,000\,q^{91} + 185\,987\,q^{92} + 51\,814\,q^{93} + 15\,036\,q^{94} + 4\,583\,q^{95} + 1\,471\,q^{96} + \\
& 496\,q^{97} + 177\,q^{98} + 68\,q^{99} + 26\,q^{100} + 11\,q^{101} + 5\,q^{102} + 2\,q^{103} + q^{104} + q^{105} ) x^{15} + \\
(19\,320\,q^{15} + 311\,465\,q^{16} + 3\,042\,808\,q^{17} + 23\,118\,465\,q^{18} + 149\,685\,670\,q^{19} + 862\,572\,395\,q^{20} + \\
& 4\,528\,544\,658\,q^{21} + 21\,961\,480\,198\,q^{22} + 99\,251\,914\,621\,q^{23} + 420\,560\,727\,420\,q^{24} + \\
& 1\,678\,281\,686\,360\,q^{25} + 6\,329\,077\,092\,453\,q^{26} + 22\,618\,117\,867\,415\,q^{27} + 76\,773\,365\,086\,100\,q^{28} + \\
& 247\,999\,842\,702\,735\,q^{29} + 763\,685\,360\,909\,770\,q^{30} + 2\,245\,154\,532\,231\,552\,q^{31} + \\
& 6\,309\,879\,915\,290\,263\,q^{32} + 16\,972\,802\,610\,729\,232\,q^{33} + 43\,742\,890\,748\,220\,477\,q^{34} + \\
& 108\,119\,250\,899\,529\,124\,q^{35} + 256\,520\,586\,246\,582\,461\,q^{36} + 584\,673\,421\,992\,357\,274\,q^{37} + \\
& 1\,281\,134\,076\,961\,638\,346\,q^{38} + 2\,700\,579\,614\,533\,070\,386\,q^{39} + 5\,479\,829\,827\,952\,799\,857\,q^{40} + \\
& 10\,709\,494\,827\,604\,086\,670\,q^{41} + 20\,169\,093\,598\,306\,723\,982\,q^{42} + 36\,620\,483\,849\,338\,454\,029\,q^{43} + \\
& 64\,131\,220\,183\,281\,379\,663\,q^{44} + 108\,366\,231\,335\,871\,500\,301\,q^{45} + 176\,747\,716\,330\,872\,745\,639\,q^{46} + \\
& 278\,350\,098\,613\,104\,210\,378\,q^{47} + 423\,385\,461\,441\,565\,247\,736\,q^{48} + 622\,163\,407\,013\,949\,073\,453\,q^{49} + \\
& 883\,491\,393\,721\,262\,596\,160\,q^{50} + 1\,212\,617\,736\,381\,684\,350\,878\,q^{51} + 1\,608\,989\,506\,687\,991\,413\,608\,q^{52} + \\
& 2\,064\,259\,206\,012\,034\,697\,756\,q^{53} + 2\,561\,070\,072\,143\,092\,759\,372\,q^{54} + 3\,073\,125\,021\,226\,493\,922\,716\,q^{55} + \\
& 3\,566\,867\,715\,628\,928\,049\,061\,q^{56} + 4\,004\,789\,260\,291\,990\,621\,810\,q^{57} + 4\,349\,984\,835\,162\,465\,146\,681\,q^{58} + \\
& 4\,571\,224\,710\,742\,375\,695\,904\,q^{59} + 4\,647\,586\,298\,937\,784\,001\,491\,q^{60} + 4\,571\,700\,970\,421\,071\,914\,980\,q^{61} + \\
& 4\,350\,920\,993\,620\,381\,139\,035\,q^{62} + 4\,006\,151\,782\,717\,342\,115\,727\,q^{63} + 3\,568\,604\,944\,103\,910\,221\,700\,q^{64} + \\
& 3\,075\,167\,175\,805\,421\,897\,902\,q^{65} + 2\,563\,331\,263\,109\,749\,220\,793\,q^{66} + 2\,066\,642\,050\,873\,380\,637\,099\,q^{67} + \\
& 1\,611\,392\,192\,597\,266\,839\,355\,q^{68} + 1\,214\,942\,590\,399\,997\,790\,201\,q^{69} + 885\,653\,486\,802\,431\,529\,240\,q^{70} + \\
& 624\,097\,600\,514\,858\,383\,973\,q^{71} + 425\,050\,636\,143\,463\,019\,881\,q^{72} + 279\,729\,964\,622\,586\,702\,774\,q^{73} + \\
& 177\,848\,370\,814\,214\,728\,354\,q^{74} + 109\,211\,280\,533\,389\,667\,566\,q^{75} + 64\,755\,649\,243\,571\,203\,894\,q^{76} + \\
& 37\,064\,489\,336\,439\,080\,294\,q^{77} + 20\,472\,843\,888\,563\,310\,367\,q^{78} + 10\,909\,377\,306\,471\,851\,409\,q^{79} + \\
& 5\,606\,320\,346\,884\,302\,466\,q^{80} + 2\,777\,537\,312\,500\,496\,686\,q^{81} + 1\,326\,136\,252\,540\,580\,428\,q^{82} + \\
& 609\,958\,858\,757\,339\,331\,q^{83} + 270\,167\,159\,748\,279\,703\,q^{84} + 115\,191\,333\,382\,542\,562\,q^{85} + \\
& 47\,260\,851\,049\,663\,380\,q^{86} + 18\,651\,996\,252\,117\,066\,q^{87} + 7\,078\,696\,306\,134\,605\,q^{88} + \\
& 2\,582\,678\,922\,965\,298\,q^{89} + 905\,723\,376\,163\,265\,q^{90} + 305\,278\,273\,137\,734\,q^{91} + 98\,902\,152\,048\,065\,q^{92} + \\
& 30\,807\,136\,379\,990\,q^{93} + 9\,231\,717\,564\,150\,q^{94} + 2\,663\,871\,174\,820\,q^{95} + 741\,262\,595\,848\,q^{96} + \\
& 199\,325\,942\,242\,q^{97} + 51\,944\,608\,686\,q^{98} + 13\,169\,762\,544\,q^{99} + 3\,264\,731\,659\,q^{100} + \\
& 796\,277\,239\,q^{101} + 192\,525\,016\,q^{102} + 46\,541\,022\,q^{103} + 11\,353\,456\,q^{104} + 2\,821\,115\,q^{105} + \\
& 720\,298\,q^{106} + 190\,443\,q^{107} + 52\,496\,q^{108} + 15\,144\,q^{109} + 4\,601\,q^{110} + 1\,474\,q^{111} + \\
& 497\,q^{112} + 177\,q^{113} + 68\,q^{114} + 26\,q^{115} + 11\,q^{116} + 5\,q^{117} + 2\,q^{118} + q^{119} + q^{120} ) x^{16} + \\
(48\,629\,q^{16} + 880\,840\,q^{17} + 9\,597\,679\,q^{18} + 81\,134\,475\,q^{19} + 584\,070\,235\,q^{20} + 3\,743\,001\,968\,q^{21} + \\
& 21\,872\,619\,648\,q^{22} + 118\,219\,705\,904\,q^{23} + 596\,418\,502\,890\,q^{24} + 2\,826\,245\,703\,364\,q^{25} +
\end{aligned}$$

$$\begin{aligned}
 &12637403418903 q^{26} + 53508871585831 q^{27} + 215149072500451 q^{28} + 823402894660149 q^{29} + \\
 &3005410655080960 q^{30} + 10479890111027882 q^{31} + 34964179546197292 q^{32} + \\
 &111759155893835862 q^{33} + 342653895232471165 q^{34} + 1008812491101019168 q^{35} + \\
 &2854788877514025792 q^{36} + 7772051747899351567 q^{37} + 20372871858428135419 q^{38} + \\
 &51457832868217433374 q^{39} + 125324294848760195159 q^{40} + 294497823353305550364 q^{41} + \\
 &668111968577834234653 q^{42} + 1464113416605849309120 q^{43} + 3100835593668327754902 q^{44} + \\
 &6349876249144888287729 q^{45} + 12578335989179680499455 q^{46} + 24111682226859005541488 q^{47} + \\
 &44744563014255916056629 q^{48} + 80410262033241321563285 q^{49} + \\
 &139984481513018036868703 q^{50} + 236142280545152528949415 q^{51} + \\
 &386110319260510311513279 q^{52} + 612071145530742434044913 q^{53} + \\
 &940904415034488076173270 q^{54} + 1402924018443117934495312 q^{55} + \\
 &2029326274858900979978955 q^{56} + 2848230949788953643852653 q^{57} + \\
 &3879479256835728968412608 q^{58} + 5128720018701909152525162 q^{59} + \\
 &6581693407803871994261460 q^{60} + 8199904045281537904481391 q^{61} + \\
 &9918945800838662488792886 q^{62} + 11650512079029430075911365 q^{63} + \\
 &13288577881062315200410221 q^{64} + 14719446568052952340567857 q^{65} + \\
 &15834479151082362855242764 q^{66} + 16543587434958539283125523 q^{67} + \\
 &16787189663016572148130262 q^{68} + 16544439216788667132483821 q^{69} + \\
 &15836157010262727784177955 q^{70} + 14721897951409598794803546 q^{71} + \\
 &13291721931136733758792325 q^{72} + 11654239242866686621899761 q^{73} + \\
 &9923119936995160480575206 q^{74} + 8204367999880312728010158 q^{75} + \\
 &6586278179927593347304878 q^{76} + 513325665976673016240492 q^{77} + \\
 &3883811812063958911512921 q^{78} + 2852228430526167335236082 q^{79} + \\
 &2032891640159124992742854 q^{80} + 1405998871501593725223996 q^{81} + \\
 &943468924289889230478354 q^{82} + 614139650345485473115615 q^{83} + \\
 &387723805114268443548062 q^{84} + 237359290957779829271247 q^{85} + \\
 &140872032761501503754974 q^{86} + 81036011529363711116935 q^{87} + \\
 &45170988927909338753810 q^{88} + 24392511242291411147353 q^{89} + 12757028684092774813156 q^{90} + \\
 &6459711408722274640471 q^{91} + 3166034685645993012551 q^{92} + 1501481067608395223673 q^{93} + \\
 &688784115979707350965 q^{94} + 305533160868048962382 q^{95} + 131007118587920787850 q^{96} + \\
 &54280006496059467600 q^{97} + 21724015517923475316 q^{98} + 8395457274454767299 q^{99} + \\
 &3131891881421632423 q^{100} + 1127433670659010928 q^{101} + 391539811853465548 q^{102} + \\
 &131148369988901281 q^{103} + 42362864270230862 q^{104} + 13195353645254294 q^{105} + \\
 &3963769077036988 q^{106} + 1148593987728103 q^{107} + 321234417220343 q^{108} + \\
 &86784366378620 q^{109} + 22676300600693 q^{110} + 5741051078254 q^{111} + 1411765942924 q^{112} + \\
 &338296078099 q^{113} + 79325102560 q^{114} + 18296525745 q^{115} + 4177279654 q^{116} + \\
 &950868855 q^{117} + 217511949 q^{118} + 50411412 q^{119} + 11932173 q^{120} + 2905512 q^{121} + \\
 &732472 q^{122} + 192218 q^{123} + 52763 q^{124} + 15186 q^{125} + 4608 q^{126} + 1475 q^{127} + \\
 &497 q^{128} + 177 q^{129} + 68 q^{130} + 26 q^{131} + 11 q^{132} + 5 q^{133} + 2 q^{134} + q^{135} + q^{136} ) x^{17} + \\
 &(123867 q^{17} + 2497405 q^{18} + 30134509 q^{19} + 281454170 q^{20} + 2236622385 q^{21} + \\
 &15822140019 q^{22} + 102120272749 q^{23} + 610230786880 q^{24} + 3407994560102 q^{25} + \\
 &17903411689892 q^{26} + 88889957079123 q^{27} + 418617683869058 q^{28} + 1875340614436735 q^{29} + \\
 &8010675872441072 q^{30} + 32692889274350103 q^{31} + 127697179143259259 q^{32} + \\
 &478089202959492921 q^{33} + 1717979587035188459 q^{34} + 5932393907656280767 q^{35} + \\
 &19706802208869010859 q^{36} + 63038411161244806000 q^{37} + 194352165367024105109 q^{38} + \\
 &578001133691694667563 q^{39} + 1659415565590706071239 q^{40} + 4602308026506445107155 q^{41} + \\
 &12338848268204902301871 q^{42} + 31997495771394273774356 q^{43} + 80305497663049036015397 q^{44} + \\
 &195160117366234303060584 q^{45} + 459479814699501949399327 q^{46} +
 \end{aligned}$$

$$\begin{aligned}
& 1\,048\,504\,161\,527\,700\,455\,909\,377\,q^{47} + 2\,319\,999\,100\,390\,533\,502\,979\,973\,q^{48} + \\
& 4\,979\,580\,592\,030\,839\,956\,458\,947\,q^{49} + 10\,371\,629\,709\,956\,292\,767\,647\,251\,q^{50} + \\
& 20\,970\,170\,037\,622\,035\,624\,390\,539\,q^{51} + 41\,171\,734\,876\,588\,410\,915\,604\,130\,q^{52} + \\
& 78\,518\,315\,917\,315\,022\,196\,658\,928\,q^{53} + 145\,492\,650\,100\,209\,964\,854\,124\,206\,q^{54} + \\
& 262\,014\,275\,417\,585\,588\,152\,816\,325\,q^{55} + 458\,701\,939\,465\,205\,244\,038\,269\,097\,q^{56} + \\
& 780\,833\,806\,066\,489\,705\,068\,028\,205\,q^{57} + 1\,292\,715\,358\,315\,119\,076\,307\,184\,560\,q^{58} + \\
& 2\,081\,856\,880\,665\,036\,957\,259\,864\,130\,q^{59} + 3\,261\,998\,351\,860\,962\,933\,524\,415\,535\,q^{60} + \\
& 4\,973\,669\,227\,215\,322\,583\,245\,101\,404\,q^{61} + 7\,380\,744\,205\,184\,597\,528\,158\,504\,182\,q^{62} + \\
& 10\,661\,491\,065\,346\,487\,301\,872\,307\,284\,q^{63} + 14\,993\,036\,043\,620\,962\,783\,914\,846\,196\,q^{64} + \\
& 20\,529\,077\,233\,041\,347\,736\,764\,812\,608\,q^{65} + 27\,372\,033\,734\,588\,440\,368\,237\,222\,206\,q^{66} + \\
& 35\,542\,458\,893\,032\,925\,791\,289\,308\,913\,q^{67} + 44\,950\,148\,303\,689\,032\,766\,583\,418\,637\,q^{68} + \\
& 55\,372\,505\,680\,622\,206\,752\,153\,079\,037\,q^{69} + 66\,445\,946\,721\,905\,842\,945\,180\,611\,660\,q^{70} + \\
& 77\,675\,097\,168\,743\,448\,546\,203\,365\,385\,q^{71} + 88\,462\,204\,883\,081\,286\,565\,256\,696\,807\,q^{72} + \\
& 98\,155\,807\,347\,405\,797\,438\,707\,013\,507\,q^{73} + 106\,113\,885\,888\,637\,629\,940\,961\,526\,198\,q^{74} + \\
& 111\,773\,335\,489\,755\,905\,145\,258\,156\,954\,q^{75} + 114\,715\,448\,859\,703\,502\,223\,876\,433\,517\,q^{76} + \\
& 114\,716\,901\,953\,374\,968\,257\,425\,111\,039\,q^{77} + 111\,777\,656\,275\,468\,661\,051\,026\,066\,748\,q^{78} + \\
& 106\,120\,957\,206\,346\,270\,223\,944\,877\,681\,q^{79} + 98\,165\,429\,383\,286\,130\,311\,882\,231\,739\,q^{80} + \\
& 88\,474\,091\,646\,930\,725\,872\,113\,343\,676\,q^{81} + 77\,688\,877\,103\,466\,725\,841\,272\,881\,081\,q^{82} + \\
& 66\,461\,170\,838\,015\,549\,419\,169\,854\,037\,q^{83} + 55\,388\,665\,221\,647\,655\,181\,814\,660\,393\,q^{84} + \\
& 44\,966\,701\,811\,981\,195\,149\,827\,815\,572\,q^{85} + 35\,558\,866\,238\,690\,899\,081\,741\,788\,464\,q^{86} + \\
& 27\,387\,792\,816\,863\,159\,149\,603\,199\,633\,q^{87} + 20\,543\,758\,153\,435\,479\,637\,946\,017\,690\,q^{88} + \\
& 15\,006\,307\,805\,303\,406\,676\,323\,970\,322\,q^{89} + 10\,673\,137\,179\,676\,092\,349\,581\,880\,591\,q^{90} + \\
& 7\,390\,665\,616\,901\,391\,214\,807\,012\,298\,q^{91} + 4\,981\,875\,188\,455\,835\,413\,196\,165\,329\,q^{92} + \\
& 3\,268\,587\,767\,584\,364\,949\,629\,606\,717\,q^{93} + 2\,086\,993\,839\,188\,392\,247\,088\,730\,182\,q^{94} + \\
& 1\,296\,602\,893\,961\,860\,214\,461\,646\,015\,q^{95} + 783\,689\,485\,775\,044\,321\,143\,053\,565\,q^{96} + \\
& 460\,737\,860\,325\,263\,299\,651\,246\,021\,q^{97} + 263\,422\,821\,272\,330\,958\,825\,072\,242\,q^{98} + \\
& 146\,438\,180\,985\,833\,626\,052\,211\,668\,q^{99} + 79\,134\,066\,548\,830\,731\,069\,971\,099\,q^{100} + \\
& 41\,560\,674\,555\,676\,467\,374\,309\,857\,q^{101} + 21\,208\,416\,155\,208\,177\,529\,067\,927\,q^{102} + \\
& 10\,513\,126\,876\,806\,615\,494\,230\,815\,q^{103} + 5\,061\,042\,453\,764\,462\,010\,277\,108\,q^{104} + \\
& 2\,365\,450\,385\,783\,560\,467\,705\,290\,q^{105} + 1\,073\,074\,894\,351\,812\,117\,759\,732\,q^{106} + \\
& 472\,346\,282\,603\,686\,687\,257\,389\,q^{107} + 201\,684\,711\,595\,767\,059\,937\,494\,q^{108} + \\
& 83\,508\,659\,469\,474\,013\,484\,316\,q^{109} + 33\,519\,474\,540\,812\,944\,503\,992\,q^{110} + \\
& 13\,038\,546\,920\,676\,473\,665\,862\,q^{111} + 4\,913\,444\,066\,107\,678\,607\,394\,q^{112} + \\
& 1\,793\,194\,264\,516\,599\,042\,965\,q^{113} + 633\,601\,653\,296\,786\,260\,788\,q^{114} + 216\,681\,840\,984\,954\,190\,481\,q^{115} + \\
& 71\,701\,132\,613\,068\,344\,077\,q^{116} + 22\,952\,094\,444\,014\,771\,415\,q^{117} + 7\,106\,062\,209\,512\,104\,525\,q^{118} + \\
& 2\,127\,620\,200\,261\,126\,433\,q^{119} + 616\,037\,183\,128\,090\,760\,q^{120} + 172\,509\,020\,261\,499\,494\,q^{121} + \\
& 46\,733\,208\,420\,006\,279\,q^{122} + 12\,253\,485\,515\,856\,675\,q^{123} + 3\,112\,054\,241\,471\,479\,q^{124} + \\
& 766\,437\,448\,887\,092\,q^{125} + 183\,329\,413\,925\,785\,q^{126} + 42\,681\,362\,094\,123\,q^{127} + 9\,698\,621\,668\,345\,q^{128} + \\
& 2\,158\,737\,372\,262\,q^{129} + 472\,752\,601\,462\,q^{130} + 102\,405\,806\,791\,q^{131} + 22\,077\,090\,164\,q^{132} + \\
& 4\,769\,060\,219\,q^{133} + 1\,039\,651\,293\,q^{134} + 230\,341\,715\,q^{135} + 52\,211\,411\,q^{136} + 12\,180\,208\,q^{137} + \\
& 2\,939\,612\,q^{138} + 737\,248\,q^{139} + 192\,917\,q^{140} + 52\,872\,q^{141} + 15\,204\,q^{142} + 4\,611\,q^{143} + 1\,476\,q^{144} + \\
& 497\,q^{145} + 177\,q^{146} + 68\,q^{147} + 26\,q^{148} + 11\,q^{149} + 5\,q^{150} + 2\,q^{151} + q^{152} + q^{153} ) x^{18} + \\
& (317\,955\,q^{18} + 7\,093\,751\,q^{19} + 94\,218\,306\,q^{20} + 966\,388\,692\,q^{21} + 8\,424\,917\,858\,q^{22} + \\
& 65\,369\,433\,943\,q^{23} + 462\,920\,304\,736\,q^{24} + 3\,037\,320\,132\,904\,q^{25} + 18\,643\,727\,197\,754\,q^{26} + \\
& 107\,776\,305\,654\,226\,q^{27} + 589\,610\,321\,947\,566\,q^{28} + 3\,063\,816\,688\,707\,384\,q^{29} + \\
& 15\,166\,666\,172\,807\,879\,q^{30} + 71\,695\,364\,611\,249\,330\,q^{31} + 324\,297\,546\,891\,224\,647\,q^{32} + \\
& 1\,406\,053\,785\,334\,292\,108\,q^{33} + 5\,852\,271\,087\,039\,072\,538\,q^{34} + 23\,415\,000\,281\,150\,521\,489\,q^{35} + \\
& 90\,163\,955\,110\,889\,365\,940\,q^{36} + 334\,512\,266\,999\,149\,363\,356\,q^{37} + 1\,196\,906\,261\,280\,653\,218\,383\,q^{38} +
\end{aligned}$$

$$\begin{aligned}
 &4\ 133\ 990\ 921\ 873\ 124\ 141\ 823\ q^{39} + 13\ 794\ 340\ 232\ 074\ 709\ 316\ 844\ q^{40} + 44\ 502\ 812\ 467\ 277\ 896\ 263\ 630\ q^{41} + \\
 &138\ 911\ 323\ 194\ 078\ 102\ 205\ 080\ q^{42} + 419\ 795\ 851\ 733\ 410\ 275\ 928\ 126\ q^{43} + \\
 &1\ 229\ 011\ 211\ 443\ 295\ 629\ 055\ 006\ q^{44} + 3\ 487\ 705\ 210\ 919\ 600\ 877\ 655\ 639\ q^{45} + \\
 &9\ 598\ 925\ 849\ 868\ 176\ 801\ 990\ 794\ q^{46} + 25\ 634\ 345\ 957\ 016\ 940\ 908\ 831\ 099\ q^{47} + \\
 &66\ 457\ 295\ 845\ 660\ 925\ 533\ 118\ 522\ q^{48} + 167\ 330\ 590\ 888\ 274\ 799\ 999\ 101\ 130\ q^{49} + \\
 &409\ 354\ 916\ 377\ 896\ 248\ 309\ 844\ 997\ q^{50} + 973\ 387\ 003\ 675\ 212\ 079\ 259\ 699\ 554\ q^{51} + \\
 &2\ 250\ 560\ 703\ 025\ 302\ 714\ 735\ 220\ 437\ q^{52} + 5\ 061\ 333\ 566\ 665\ 424\ 240\ 716\ 340\ 041\ q^{53} + \\
 &11\ 075\ 130\ 107\ 876\ 168\ 126\ 480\ 346\ 214\ q^{54} + 23\ 587\ 111\ 223\ 414\ 182\ 760\ 470\ 611\ 584\ q^{55} + \\
 &48\ 906\ 593\ 260\ 040\ 741\ 113\ 446\ 494\ 604\ q^{56} + 98\ 751\ 631\ 278\ 575\ 024\ 347\ 421\ 438\ 905\ q^{57} + \\
 &194\ 229\ 874\ 572\ 315\ 970\ 367\ 559\ 117\ 840\ q^{58} + 372\ 209\ 056\ 687\ 548\ 075\ 636\ 370\ 754\ 073\ q^{59} + \\
 &695\ 112\ 701\ 608\ 353\ 751\ 369\ 837\ 198\ 868\ q^{60} + 1\ 265\ 357\ 966\ 816\ 921\ 752\ 979\ 971\ 116\ 847\ q^{61} + \\
 &2\ 245\ 683\ 608\ 367\ 390\ 183\ 370\ 361\ 237\ 782\ q^{62} + 3\ 886\ 356\ 833\ 312\ 033\ 542\ 025\ 244\ 984\ 295\ q^{63} + \\
 &6\ 559\ 530\ 900\ 034\ 723\ 998\ 412\ 591\ 965\ 236\ q^{64} + 10\ 799\ 690\ 258\ 706\ 203\ 794\ 750\ 055\ 182\ 450\ q^{65} + \\
 &17\ 347\ 072\ 053\ 617\ 850\ 752\ 966\ 201\ 896\ 076\ q^{66} + 27\ 188\ 242\ 089\ 597\ 464\ 470\ 855\ 820\ 268\ 832\ q^{67} + \\
 &41\ 584\ 908\ 821\ 299\ 724\ 369\ 132\ 881\ 822\ 203\ q^{68} + 62\ 079\ 148\ 490\ 320\ 417\ 375\ 800\ 191\ 037\ 251\ q^{69} + \\
 &90\ 461\ 344\ 185\ 295\ 020\ 287\ 062\ 636\ 486\ 847\ q^{70} + 128\ 687\ 356\ 480\ 291\ 002\ 849\ 577\ 392\ 861\ 342\ q^{71} + \\
 &178\ 734\ 760\ 919\ 910\ 346\ 876\ 633\ 713\ 567\ 739\ q^{72} + 242\ 395\ 077\ 269\ 094\ 443\ 306\ 257\ 098\ 128\ 223\ q^{73} + \\
 &321\ 009\ 734\ 671\ 316\ 888\ 751\ 431\ 439\ 257\ 168\ q^{74} + 415\ 171\ 010\ 797\ 266\ 072\ 133\ 440\ 597\ 514\ 545\ q^{75} + \\
 &524\ 423\ 176\ 876\ 020\ 700\ 169\ 099\ 728\ 883\ 137\ q^{76} + 647\ 010\ 478\ 880\ 646\ 394\ 510\ 103\ 208\ 405\ 192\ q^{77} + \\
 &779\ 723\ 926\ 606\ 123\ 272\ 139\ 890\ 499\ 680\ 942\ q^{78} + 917\ 895\ 167\ 151\ 818\ 092\ 411\ 745\ 948\ 956\ 685\ q^{79} + \\
 &1\ 055\ 571\ 447\ 067\ 780\ 055\ 061\ 247\ 859\ 578\ 027\ q^{80} + 1\ 185\ 881\ 508\ 659\ 839\ 836\ 149\ 950\ 440\ 019\ 601\ q^{81} + \\
 &1\ 301\ 571\ 473\ 640\ 770\ 954\ 631\ 564\ 995\ 719\ 045\ q^{82} + 1\ 395\ 657\ 819\ 567\ 519\ 145\ 414\ 663\ 458\ 380\ 403\ q^{83} + \\
 &1\ 462\ 118\ 117\ 346\ 133\ 883\ 125\ 963\ 154\ 354\ 156\ q^{84} + 1\ 496\ 525\ 582\ 073\ 834\ 189\ 276\ 226\ 207\ 348\ 733\ q^{85} + \\
 &1\ 496\ 534\ 990\ 156\ 260\ 530\ 803\ 511\ 592\ 430\ 392\ q^{86} + 1\ 462\ 146\ 118\ 998\ 498\ 611\ 331\ 954\ 686\ 163\ 171\ q^{87} + \\
 &1\ 395\ 703\ 738\ 411\ 615\ 431\ 469\ 720\ 737\ 828\ 170\ q^{88} + 1\ 301\ 634\ 158\ 489\ 993\ 957\ 470\ 052\ 015\ 438\ 291\ q^{89} + \\
 &1\ 185\ 959\ 313\ 016\ 087\ 799\ 639\ 602\ 021\ 766\ 279\ q^{90} + 1\ 055\ 662\ 226\ 697\ 523\ 813\ 203\ 387\ 637\ 274\ 194\ q^{91} + \\
 &917\ 996\ 312\ 960\ 383\ 679\ 367\ 494\ 103\ 799\ 932\ q^{92} + 779\ 832\ 444\ 687\ 263\ 615\ 221\ 323\ 570\ 551\ 131\ q^{93} + \\
 &647\ 123\ 120\ 506\ 494\ 039\ 038\ 373\ 048\ 429\ 785\ q^{94} + 524\ 536\ 610\ 512\ 269\ 866\ 185\ 640\ 673\ 708\ 648\ q^{95} + \\
 &415\ 282\ 018\ 571\ 313\ 323\ 894\ 540\ 539\ 455\ 904\ q^{96} + 321\ 115\ 409\ 398\ 025\ 887\ 315\ 918\ 232\ 576\ 405\ q^{97} + \\
 &242\ 492\ 994\ 601\ 720\ 404\ 807\ 381\ 728\ 273\ 599\ q^{98} + 178\ 823\ 104\ 703\ 352\ 179\ 587\ 755\ 800\ 926\ 690\ q^{99} + \\
 &128\ 764\ 983\ 005\ 773\ 120\ 515\ 879\ 006\ 486\ 942\ q^{100} + 90\ 527\ 780\ 957\ 713\ 648\ 947\ 765\ 724\ 949\ 206\ q^{101} + \\
 &62\ 134\ 533\ 126\ 669\ 661\ 400\ 490\ 238\ 826\ 045\ q^{102} + 41\ 629\ 881\ 796\ 571\ 384\ 940\ 987\ 260\ 941\ 741\ q^{103} + \\
 &27\ 223\ 811\ 876\ 790\ 838\ 684\ 338\ 255\ 096\ 079\ q^{104} + 17\ 374\ 472\ 289\ 044\ 828\ 232\ 369\ 242\ 677\ 394\ q^{105} + \\
 &10\ 820\ 246\ 285\ 895\ 070\ 410\ 392\ 283\ 795\ 906\ q^{106} + 6\ 574\ 548\ 419\ 326\ 813\ 364\ 883\ 324\ 307\ 008\ q^{107} + \\
 &3\ 897\ 039\ 709\ 736\ 668\ 503\ 506\ 663\ 474\ 621\ q^{108} + 2\ 253\ 082\ 406\ 067\ 602\ 606\ 942\ 641\ 898\ 396\ q^{109} + \\
 &1\ 270\ 346\ 402\ 277\ 850\ 097\ 612\ 710\ 854\ 926\ q^{110} + 698\ 386\ 414\ 926\ 758\ 506\ 886\ 289\ 528\ 973\ q^{111} + \\
 &374\ 299\ 933\ 367\ 638\ 348\ 102\ 617\ 275\ 258\ q^{112} + 195\ 529\ 330\ 865\ 374\ 631\ 644\ 755\ 156\ 160\ q^{113} + \\
 &99\ 537\ 355\ 238\ 705\ 845\ 007\ 523\ 039\ 027\ q^{114} + 49\ 368\ 738\ 523\ 886\ 424\ 403\ 259\ 122\ 094\ q^{115} + \\
 &23\ 851\ 478\ 588\ 460\ 879\ 515\ 198\ 593\ 536\ q^{116} + 11\ 222\ 183\ 182\ 127\ 987\ 033\ 849\ 487\ 102\ q^{117} + \\
 &5\ 140\ 855\ 849\ 290\ 902\ 532\ 099\ 443\ 062\ q^{118} + 2\ 292\ 359\ 036\ 398\ 229\ 295\ 805\ 541\ 194\ q^{119} + \\
 &994\ 736\ 459\ 635\ 724\ 605\ 365\ 124\ 125\ q^{120} + 419\ 949\ 163\ 328\ 780\ 149\ 772\ 596\ 875\ q^{121} + \\
 &172\ 436\ 839\ 355\ 030\ 963\ 536\ 956\ 134\ q^{122} + 68\ 847\ 147\ 828\ 461\ 174\ 315\ 094\ 643\ q^{123} + \\
 &26\ 720\ 175\ 180\ 151\ 866\ 963\ 809\ 929\ q^{124} + 10\ 077\ 725\ 213\ 092\ 293\ 807\ 245\ 461\ q^{125} + \\
 &3\ 692\ 549\ 206\ 176\ 800\ 151\ 464\ 303\ q^{126} + 1\ 314\ 015\ 943\ 330\ 818\ 665\ 609\ 232\ q^{127} + \\
 &454\ 000\ 305\ 399\ 598\ 791\ 527\ 230\ q^{128} + 152\ 252\ 957\ 748\ 516\ 942\ 612\ 192\ q^{129} + \\
 &49\ 545\ 800\ 497\ 249\ 814\ 341\ 783\ q^{130} + 15\ 640\ 991\ 174\ 041\ 408\ 720\ 451\ q^{131} + \\
 &4\ 788\ 878\ 152\ 479\ 893\ 577\ 181\ q^{132} + 1\ 421\ 761\ 732\ 535\ 733\ 418\ 302\ q^{133} + 409\ 238\ 375\ 864\ 253\ 296\ 079\ q^{134} + \\
 &114\ 194\ 303\ 826\ 237\ 480\ 132\ q^{135} + 30\ 890\ 984\ 828\ 371\ 706\ 391\ q^{136} + 8\ 101\ 950\ 589\ 023\ 379\ 196\ q^{137} +
 \end{aligned}$$

$$\begin{aligned}
& 2\,060\,795\,856\,689\,656\,490\,q^{138} + 508\,591\,905\,933\,448\,208\,q^{139} + 121\,871\,516\,176\,095\,862\,q^{140} + \\
& 28\,384\,299\,535\,713\,614\,q^{141} + 6\,434\,443\,990\,439\,781\,q^{142} + 1\,422\,396\,924\,118\,562\,q^{143} + \\
& 307\,378\,986\,623\,291\,q^{144} + 65\,136\,315\,811\,217\,q^{145} + 13\,587\,381\,715\,854\,q^{146} + 2\,802\,900\,901\,339\,q^{147} + \\
& 574\,836\,382\,320\,q^{148} + 117\,896\,319\,073\,q^{149} + 24\,331\,970\,786\,q^{150} + 5\,084\,972\,066\,q^{151} + \\
& 1\,082\,481\,188\,q^{152} + 236\,007\,972\,q^{153} + 52\,951\,701\,q^{154} + 12\,277\,230\,q^{155} + 2\,952\,621\,q^{156} + \\
& 739\,070\,q^{157} + 193\,186\,q^{158} + 52\,914\,q^{159} + 15\,211\,q^{160} + 4\,612\,q^{161} + 1\,476\,q^{162} + \\
& 497\,q^{163} + 177\,q^{164} + 68\,q^{165} + 26\,q^{166} + 11\,q^{167} + 5\,q^{168} + 2\,q^{169} + q^{170} + q^{171} ) x^{19} + \\
& (823\,065\,q^{19} + 20\,187\,313\,q^{20} + 293\,509\,092\,q^{21} + 3\,288\,208\,176\,q^{22} + 31\,277\,128\,998\,q^{23} + \\
& 264\,691\,559\,084\,q^{24} + 2\,044\,794\,578\,470\,q^{25} + 14\,643\,492\,796\,356\,q^{26} + 98\,184\,235\,219\,447\,q^{27} + \\
& 620\,596\,419\,754\,320\,q^{28} + 3\,716\,238\,143\,794\,229\,q^{29} + 21\,162\,430\,045\,179\,679\,q^{30} + \\
& 114\,945\,806\,126\,821\,067\,q^{31} + 596\,960\,878\,417\,732\,999\,q^{32} + 2\,970\,376\,081\,489\,647\,292\,q^{33} + \\
& 14\,185\,655\,872\,839\,373\,515\,q^{34} + 65\,121\,007\,644\,834\,038\,062\,q^{35} + 287\,746\,758\,737\,092\,907\,820\,q^{36} + \\
& 1\,225\,290\,373\,652\,216\,707\,483\,q^{37} + 5\,033\,583\,407\,520\,297\,279\,028\,q^{38} + 19\,968\,836\,535\,449\,084\,168\,383\,q^{39} + \\
& 76\,569\,651\,362\,948\,734\,829\,134\,q^{40} + 284\,019\,909\,750\,895\,000\,170\,551\,q^{41} + \\
& 1\,019\,909\,235\,728\,754\,468\,550\,479\,q^{42} + 3\,548\,159\,746\,827\,992\,081\,226\,284\,q^{43} + \\
& 11\,966\,316\,845\,647\,258\,474\,019\,835\,q^{44} + 39\,147\,230\,539\,523\,542\,307\,707\,538\,q^{45} + \\
& 124\,301\,081\,973\,626\,347\,318\,503\,939\,q^{46} + 383\,280\,196\,746\,760\,526\,410\,176\,411\,q^{47} + \\
& 1\,148\,273\,372\,101\,180\,442\,469\,734\,182\,q^{48} + 3\,344\,015\,064\,897\,641\,205\,935\,721\,820\,q^{49} + \\
& 9\,470\,644\,674\,318\,829\,108\,947\,465\,895\,q^{50} + 26\,095\,302\,356\,174\,027\,362\,225\,106\,635\,q^{51} + \\
& 69\,982\,464\,159\,630\,126\,652\,555\,257\,244\,q^{52} + 182\,735\,597\,473\,752\,285\,342\,186\,915\,172\,q^{53} + \\
& 464\,748\,499\,901\,057\,959\,634\,241\,287\,659\,q^{54} + 1\,151\,646\,506\,970\,846\,841\,565\,289\,311\,387\,q^{55} + \\
& 2\,781\,405\,533\,840\,678\,398\,014\,826\,292\,552\,q^{56} + 6\,549\,128\,871\,065\,795\,489\,406\,109\,767\,011\,q^{57} + \\
& 15\,038\,347\,569\,821\,431\,335\,185\,496\,629\,909\,q^{58} + 33\,684\,554\,978\,016\,782\,559\,275\,388\,816\,023\,q^{59} + \\
& 73\,618\,380\,798\,722\,025\,259\,996\,756\,038\,875\,q^{60} + 157\,025\,914\,947\,995\,135\,561\,555\,587\,971\,707\,q^{61} + \\
& 326\,953\,103\,173\,466\,735\,243\,149\,892\,835\,822\,q^{62} + 664\,695\,824\,717\,663\,004\,561\,665\,567\,503\,220\,q^{63} + \\
& 1\,319\,693\,953\,936\,285\,751\,269\,400\,494\,647\,682\,q^{64} + 2\,559\,299\,525\,847\,383\,595\,184\,250\,339\,437\,262\,q^{65} + \\
& 4\,848\,942\,860\,304\,840\,953\,209\,526\,262\,981\,561\,q^{66} + 8\,976\,920\,217\,975\,220\,242\,622\,389\,150\,678\,837\,q^{67} + \\
& 16\,241\,806\,753\,109\,061\,084\,104\,027\,741\,669\,085\,q^{68} + 28\,723\,445\,443\,568\,031\,358\,362\,829\,070\,122\,398\,q^{69} + \\
& 49\,659\,104\,556\,050\,789\,103\,088\,297\,831\,701\,406\,q^{70} + 83\,942\,687\,791\,176\,352\,104\,200\,536\,910\,613\,804\,q^{71} + \\
& 138\,754\,366\,759\,890\,586\,880\,736\,351\,212\,328\,170\,q^{72} + 224\,308\,243\,636\,208\,620\,690\,118\,080\,959\,540\,349\,q^{73} + \\
& 354\,674\,952\,264\,755\,966\,163\,609\,266\,995\,457\,991\,q^{74} + 548\,594\,545\,183\,702\,698\,009\,102\,832\,502\,272\,198\,q^{75} + \\
& 830\,145\,575\,465\,460\,556\,042\,555\,828\,342\,320\,439\,q^{76} + 1\,229\,085\,815\,977\,693\,091\,488\,153\,681\,557\,838\,661\,q^{77} + \\
& 1\,780\,639\,629\,188\,274\,269\,517\,943\,381\,508\,926\,516\,q^{78} + 2\,524\,491\,153\,296\,134\,571\,596\,765\,233\,220\,612\,112\,q^{79} + \\
& 3\,502\,767\,059\,145\,202\,733\,647\,777\,392\,422\,183\,107\,q^{80} + 4\,756\,871\,160\,348\,002\,273\,394\,632\,842\,699\,036\,344\,q^{81} + \\
& 6\,323\,172\,067\,609\,348\,742\,051\,107\,143\,134\,394\,045\,q^{82} + 8\,227\,738\,977\,078\,195\,638\,112\,534\,241\,630\,306\,376\,q^{83} + \\
& 10\,480\,548\,972\,933\,107\,953\,932\,016\,281\,719\,054\,361\,q^{84} + \\
& 13\,069\,815\,705\,079\,621\,743\,921\,003\,716\,939\,108\,393\,q^{85} + \\
& 15\,957\,265\,987\,523\,493\,030\,974\,397\,160\,111\,658\,515\,q^{86} + 19\,075\,266\,320\,852\,141\,389\,638\,418\,667\,347\,092\,506\, \\
& q^{87} + 22\,326\,632\,536\,537\,736\,246\,250\,432\,116\,344\,981\,477\,q^{88} + \\
& 25\,587\,720\,949\,261\,299\,370\,358\,099\,679\,362\,505\,118\,q^{89} + 28\,715\,008\,657\,621\,083\,900\,907\,012\,739\,713\,092\,025\, \\
& q^{90} + 31\,554\,870\,395\,449\,515\,808\,578\,242\,767\,226\,971\,466\,q^{91} + \\
& 33\,955\,728\,428\,897\,592\,161\,022\,178\,750\,285\,127\,380\,q^{92} + 35\,781\,288\,549\,070\,202\,292\,490\,326\,089\,549\,246\,191\, \\
& q^{93} + 36\,923\,276\,574\,731\,685\,070\,296\,807\,055\,707\,097\,466\,q^{94} + \\
& 37\,312\,029\,879\,842\,732\,285\,272\,430\,516\,653\,726\,624\,q^{95} + 36\,923\,508\,419\,108\,918\,667\,264\,721\,876\,770\,133\,733\, \\
& q^{96} + 35\,781\,747\,273\,111\,549\,844\,843\,422\,145\,210\,073\,457\,q^{97} + \\
& 33\,956\,403\,943\,864\,377\,210\,887\,966\,667\,034\,143\,560\,q^{98} + 31\,555\,747\,252\,225\,540\,339\,592\,163\,576\,053\,604\,715\, \\
& q^{99} + 28\,716\,065\,877\,405\,130\,222\,593\,453\,057\,481\,649\,101\,q^{100} + \\
& 25\,588\,932\,091\,068\,494\,306\,765\,921\,202\,932\,341\,494\,q^{101} + 22\,327\,966\,153\,612\,307\,499\,828\,378\,274\,650\,054\,485
\end{aligned}$$



$$\begin{aligned}
 & q^{102} + 19\,076\,686\,900\,005\,127\,822\,175\,630\,370\,083\,874\,929\, q^{103} + \\
 & 15\,958\,735\,379\,213\,146\,488\,663\,636\,447\,327\,320\,135\, q^{104} + 13\,071\,294\,956\,539\,529\,792\,818\,555\,104\,477\,722\,094\, \\
 & q^{105} + 10\,482\,000\,392\,653\,917\,112\,648\,977\,449\,181\,628\,273\, q^{106} + \\
 & 8\,229\,128\,204\,502\,842\,953\,443\,500\,329\,417\,904\,216\, q^{107} + 6\,324\,469\,908\,212\,809\,274\,402\,693\,671\,989\,209\,656\, \\
 & q^{108} + 4\,758\,054\,975\,423\,667\,152\,948\,919\,811\,585\,905\,564\, q^{109} + \\
 & 3\,503\,821\,564\,546\,040\,571\,133\,903\,267\,355\,667\,142\, q^{110} + 2\,525\,408\,567\,993\,894\,937\,622\,530\,197\,449\,561\,805\, \\
 & q^{111} + 1\,781\,419\,205\,333\,296\,591\,894\,840\,360\,242\,739\,830\, q^{112} + \\
 & 1\,229\,732\,860\,389\,436\,464\,277\,514\,439\,659\,319\,394\, q^{113} + 830\,670\,125\,630\,134\,995\,066\,211\,451\,110\,698\,815\, q^{114} + \\
 & 549\,009\,884\,756\,060\,888\,420\,423\,282\,565\,655\,039\, q^{115} + 354\,996\,142\,467\,240\,399\,873\,257\,672\,075\,279\,140\, q^{116} + \\
 & 224\,550\,814\,164\,821\,404\,422\,077\,092\,162\,349\,235\, q^{117} + 138\,933\,262\,577\,665\,206\,444\,745\,763\,421\,123\,679\, q^{118} + \\
 & 84\,071\,517\,040\,249\,721\,640\,638\,652\,992\,587\,862\, q^{119} + 49\,749\,686\,789\,686\,697\,549\,692\,761\,919\,310\,326\, q^{120} + \\
 & 28\,785\,624\,560\,501\,251\,288\,712\,341\,142\,322\,151\, q^{121} + 16\,283\,472\,046\,018\,958\,295\,273\,719\,578\,449\,823\, q^{122} + \\
 & 9\,004\,171\,366\,134\,827\,513\,763\,252\,879\,388\,036\, q^{123} + 4\,866\,337\,862\,285\,006\,231\,497\,837\,428\,392\,140\, q^{124} + \\
 & 2\,570\,134\,777\,765\,225\,677\,014\,388\,306\,146\,707\, q^{125} + 1\,326\,279\,178\,777\,066\,991\,303\,889\,335\,017\,704\, q^{126} + \\
 & 668\,600\,258\,872\,272\,520\,452\,382\,702\,669\,711\, q^{127} + 329\,211\,170\,618\,787\,874\,497\,279\,907\,231\,118\, q^{128} + \\
 & 158\,299\,532\,266\,059\,010\,755\,996\,600\,872\,085\, q^{129} + 74\,318\,855\,786\,911\,002\,110\,024\,443\,781\,018\, q^{130} + \\
 & 34\,060\,152\,515\,535\,562\,739\,333\,878\,609\,685\, q^{131} + 15\,234\,661\,182\,309\,216\,448\,682\,422\,629\,577\, q^{132} + \\
 & 6\,649\,127\,286\,481\,262\,216\,627\,273\,103\,673\, q^{133} + 2\,831\,037\,851\,475\,071\,265\,815\,055\,774\,736\, q^{134} + \\
 & 1\,175\,644\,485\,605\,370\,770\,902\,605\,755\,065\, q^{135} + 476\,049\,830\,528\,925\,068\,630\,045\,925\,681\, q^{136} + \\
 & 187\,918\,006\,141\,834\,961\,268\,461\,103\,197\, q^{137} + 72\,296\,017\,217\,107\,605\,599\,092\,509\,108\, q^{138} + \\
 & 27\,100\,537\,932\,147\,278\,425\,594\,550\,193\, q^{139} + 9\,895\,643\,796\,573\,961\,301\,552\,317\,747\, q^{140} + \\
 & 3\,518\,809\,554\,458\,017\,744\,697\,487\,836\, q^{141} + 1\,218\,188\,539\,173\,883\,938\,512\,147\,029\, q^{142} + \\
 & 410\,469\,648\,909\,928\,632\,776\,905\,045\, q^{143} + 134\,578\,729\,922\,875\,456\,593\,878\,712\, q^{144} + \\
 & 42\,922\,326\,428\,044\,510\,505\,628\,967\, q^{145} + 13\,313\,350\,898\,290\,506\,198\,477\,534\, q^{146} + \\
 & 4\,014\,948\,530\,988\,658\,938\,260\,090\, q^{147} + 1\,176\,956\,065\,187\,994\,565\,472\,074\, q^{148} + \\
 & 335\,304\,026\,146\,028\,837\,575\,141\, q^{149} + 92\,820\,048\,065\,060\,351\,596\,285\, q^{150} + \\
 & 24\,964\,141\,583\,827\,775\,707\,707\, q^{151} + 6\,522\,866\,463\,614\,203\,374\,206\, q^{152} + \\
 & 1\,655\,841\,933\,665\,904\,611\,888\, q^{153} + 408\,428\,773\,758\,259\,183\,757\, q^{154} + 97\,915\,078\,551\,983\,292\,498\, q^{155} + \\
 & 22\,825\,160\,249\,920\,301\,342\, q^{156} + 5\,177\,268\,406\,684\,007\,871\, q^{157} + 1\,143\,725\,640\,219\,001\,163\, q^{158} + \\
 & 246\,397\,974\,438\,984\,738\, q^{159} + 51\,854\,408\,419\,761\,129\, q^{160} + 10\,683\,644\,653\,928\,484\, q^{161} + \\
 & 2\,160\,902\,469\,407\,100\, q^{162} + 430\,526\,581\,364\,122\, q^{163} + 84\,832\,494\,208\,236\, q^{164} + 16\,608\,816\,162\,570\, q^{165} + \\
 & 3\,247\,646\,993\,041\, q^{166} + 637\,725\,171\,137\, q^{167} + 126\,457\,607\,021\, q^{168} + 25\,458\,149\,444\, q^{169} + \\
 & 5\,228\,952\,959\, q^{170} + 1\,100\,528\,507\, q^{171} + 238\,253\,067\, q^{172} + 53\,233\,295\, q^{173} + 12\,313\,532\, q^{174} + \\
 & 2\,957\,523\, q^{175} + 739\,775\, q^{176} + 193\,295\, q^{177} + 52\,932\, q^{178} + 15\,214\, q^{179} + 4\,613\, q^{180} + 1\,476\, q^{181} + \\
 & 497\, q^{182} + 177\, q^{183} + 68\, q^{184} + 26\, q^{185} + 11\, q^{186} + 5\, q^{187} + 2\, q^{188} + q^{189} + q^{190} ) x^{20}
 \end{aligned}$$