**Math 1B—Calculus, Spring 2018—Haiman**
**Practice problems for Midterm 2**

1. Can the Integral Test be used to determine whether the series

$$\sum_{n=0}^{\infty} \frac{\cos^2(n)}{n^2+1}$$

is convergent? Why or why not?

2. Determine whether the series in Problem 1 is convergent. Explain your method.

3. Given the fact that

$$\sum_{n=1}^{\infty} \frac{1}{n^2} = \pi^2/6,$$

evaluate

$$\sum_{n=3}^{\infty} \frac{1}{(n-1)^2}.$$

4. Find a value of $n$ such that the approximation to the series

$$\sum_{n=1}^{\infty} \frac{1}{n^4}$$

given by its $n$-th partial sum $s_n$ is within .01 of the exact value.

5. Determine whether each series is divergent, absolutely convergent, or conditionally convergent. Give a reason.

(a)

$$\sum_{n=1}^{\infty} \frac{n-1}{n^3+1}$$

(b)

$$\sum_{n=1}^{\infty} \frac{n\sin(n)}{n^3+1}$$

(c)

$$\sum_{n=1}^{\infty} \sin(1/n)$$

(d)

$$\sum_{n=1}^{\infty} (-1)^n \arctan(n)$$

(e)

$$\sum_{n=1}^{\infty} n\, e^{-n}$$

(f)

$$\sum_{n=1}^{\infty} (-1)^n \frac{\sqrt{n}}{2n+5}$$

(g)

$$\sum_{n=1}^{\infty} \frac{n!}{2 \cdot 4 \cdot \cdots \cdot 2n}$$

(h)

$$\sum_{n=1}^{\infty} \frac{n\pi^n}{(-3)^n}$$

6. For what values of $p$ is the series

$$\sum_{n=2}^{\infty} \frac{(\ln n)^p}{n}$$

convergent?

7. Find the radius of convergence and the interval of convergence of the series

$$\sum_{n=0}^{\infty} \frac{(-2)^n}{n+1}(x+1)^n.$$

8. Find the radius of convergence and the interval of convergence of the series

$$\sum_{n=1}^{\infty} \frac{(3x-4)^n}{n^2}.$$

9. (a) Find the interval of convergence of the series

$$1 + 2x + x^2 + 2x^3 + x^4 + 2x^5 + \cdots$$

(that is, the power series with coefficients $c_n = 1$ if $n$ is even, $c_n = 2$ if $n$ is odd).

(b) Find an explicit formula (not in the form of a series) for the function $f(x)$ that this series converges to.

10. Find a power series representation of each function and determine the interval of convergence.

(a)

$$f(x) = \frac{x+3}{x-2}$$

(b)

$$f(x) = x \arctan(x^3)$$

(c)

$$f(x) = \int_0^x e^{-t^2}\, dt$$

(d)

$$f(x) = \sqrt[3]{27 - x}$$

11. Find the first four terms in the Taylor series expansion for the function $f(x) = xe^{-x}$, centered at $x = 1$.

12. Find the sum of the series

$$\sum_{n=0}^{\infty} \frac{4^n}{5^{n+1} n!}$$

13. Use Euler's method with a step size of .1 to approximate the solution $y = f(x)$ to the initial value problem $y' = x - y^2$, $y(0) = 1$ on the interval $[0, .5]$. (You will want to use a calculator for this practice problem. It will be possible to solve the exam problems without using a calculator.)

14. Let $y = f(x)$ be the equation of a curve that passes through the point $(1, 2)$ and as slope $x/y$ at each point $(x, y)$.

(a) Find a differential equation and initial condition whose solution is $y = f(x)$.

(b) Solve to find $f(x)$.

15. The air in a room with volume 100 cubic meters initially contains .2% carbon dioxide. Fresher air containing .1% carbon dioxide flows into the room at a rate of 1 cubic meter per minute and the mixed air flows out at the same rate. Find the percentage of carbon dioxide in the air as a function of time.

16. Find the general solution of the differential equation

$$2xy' + y = 2\sqrt{x}$$

17. Find the general solution of the differential equation

$$(1 + \cos x)y' = (1 + y) \sin x$$

18. Solve the initial value problem

$$y' = 2xy^2, \quad y(1) = 1.$$