Math 1B—Calculus, Spring 2018—Haiman
Practice problems for Midterm 1

1. Evaluate the integral
   \[ \int (\ln x)^2 \, dx \]

2. Evaluate the definite integral
   \[ \int_0^1 \frac{dx}{(x^2 + 1)^{3/2}} \]

3. Evaluate the integral
   \[ \int \frac{x}{2x} \, dx \]

4. Evaluate the integral
   \[ \int \frac{x^3 + 4x^2 + x - 1}{x^3 + x^2} \, dx \]

5. Evaluate the integral
   \[ \int x \sin^2 x \, dx \]

6. Evaluate the definite integral
   \[ \int_0^4 e^{\sqrt{x}} \, dx \]

7. Evaluate the integral
   \[ \int \sin(3x) \cos(5x) \, dx \]

8. Evaluate the integral
   \[ \int \tan^2 \theta \sec^4 \theta \, d\theta \]

9. Evaluate the definite integral
   \[ \int_0^\pi x \sin x \cos x \, dx \]

10. Evaluate the integral
    \[ \int \sqrt{3 + 2x - x^2} \, dx \]
11. Evaluate the integral
\[ \int \ln(x^2 - x + 2) \, dx \]
Hint: use integration by parts to get started.

12. (a) Use Simpson’s rule with \( n = 4 \) to find a rational number (a fraction with integer numerator and denominator) which approximates the integral
\[ \int_1^2 \frac{1}{x} \, dx. \]
(b) Use a calculator to compare your answer with the exact value \( \ln(2) \) [this part would not actually be on an exam, since calculators are not allowed].

13. Explain why each of the following integrals is improper, and determine whether it is convergent or divergent:
(a) 
\[ \int_0^\infty \frac{1}{1 + x^3} \, dx \]
(b) 
\[ \int_{-1}^{0} \frac{1}{1 + x^3} \, dx \]

14. Evaluate the improper integral
\[ \int_1^\infty \frac{e^{-1/x}}{x^2} \, dx \]

15. (a) Sketch the region bounded by the \( x \) and \( y \) axes and the curve
\[ y = \frac{1}{\sqrt{x} (x + 1)}. \]
(b) Find the area of this region, if it is finite. Hint: make a substitution to evaluate the relevant integral.

16. Find the arc length of the curve
\[ y = \frac{x^3}{3} + \frac{1}{4x} \]
from \( x = 1 \) to \( x = 2 \).

17. (a) Sketch the curve \( y = x^3 \) for \( 0 \leq x \leq 1 \)
(b) Sketch the surface obtained by rotating this curve about the \( x \) axis.
(c) Find the area of the surface in part (b).
18. A cylindrical tank with a radius of 2 m is half full of water. Find the hydrostatic force (in Newtons) on one end of the tank. Use the values $\rho = 10^3$ kg/m$^3$ for the density of water and $g = 9.8$ N/kg for the acceleration of gravity.

19. Find the centroid of the triangular region with vertices at $(0, 0)$, $(3, 0)$ and $(0, 4)$.

20. (a) Find a formula for the $n$-th term $a_n$ (starting at $n = 1$) of the sequence

$$\frac{\sqrt{1\cdot 2}}{1}, \frac{\sqrt{2\cdot 3}}{3}, \frac{\sqrt{3\cdot 4}}{5}, \frac{\sqrt{4\cdot 5}}{7}, \ldots$$

(b) Find the limit $\lim_{n \to \infty} a_n$ of this sequence.

21. For each of the following, decide whether the sequence converges, diverges to $\infty$ or $-\infty$, or diverges without going to $\pm \infty$.

(a) $\lim_{n \to \infty} n \cos(\pi n)$
(b) $\lim_{n \to \infty} \cos(\pi n)/n$
(c) $\lim_{n \to \infty} n^3/2^n$
(d) $\lim_{n \to \infty} \sqrt{n} + \arctan(n^2)$

22. Express the repeating decimal $3.272727\ldots$ as a rational number $p/q$, where $p$ and $q$ are integers with no common factor.

23. Find the sum of the geometric series $2 - \frac{4}{3} + \frac{8}{9} - \frac{16}{27} + \cdots$. 