Math 1B—Calculus, Spring 2018—Haiman
Practice problems for Midterm 1

1. Evaluate the integral

$$\int (\ln x)^2 \, dx$$

2. Evaluate the definite integral

$$\int_0^1 \frac{dx}{(x^2+1)^{3/2}}$$

3. Evaluate the integral

$$\int \frac{x}{2^x} \, dx$$

4. Evaluate the integral

$$\int \frac{x^3+4x^2+x-1}{x^3+x^2} \, dx$$

5. Evaluate the integral

$$\int x \sin^2 x \, dx$$

6. Evaluate the definite integral

$$\int_0^4 e^{\sqrt{x}} \, dx$$

7. Evaluate the integral

$$\int \sin(3x)\cos(5x) \, dx$$

8. Evaluate the integral

$$\int \tan^2\theta \sec^4\theta \, d\theta$$

9. Evaluate the definite integral

$$\int_0^\pi x \sin x \cos x \, dx$$

10. Evaluate the integral

$$\int \sqrt{3+2x-x^2} \, dx$$

11. Evaluate the integral

$$\int \ln(x^2 - x + 2)\, dx$$

Hint: use integration by parts to get started.

12. (a) Use Simpson's rule with $n = 4$ to find a rational number (a fraction with integer numerator and denominator) which approximates the integral

$$\int_1^2 \frac{1}{x}\, dx.$$

(b) Use a calculator to compare your answer with the exact value $\ln(2)$ [this part would not actually be on an exam, since calculators are not allowed].

13. Explain why each of the following integrals is improper, and determine whether it is convergent or divergent:

(a)

$$\int_0^\infty \frac{1}{1+x^3}\, dx$$

(b)

$$\int_{-1}^0 \frac{1}{1+x^3}\, dx$$

14. Evaluate the improper integral

$$\int_1^\infty \frac{e^{-1/x}}{x^2}\, dx$$

15. (a) Sketch the region bounded by the x and y axes and the curve

$$y = \frac{1}{\sqrt{x}\,(x+1)}.$$

(b) Find the area of this region, if it is finite. Hint: make a substitution to evaluate the relevant integral.

16. Find the arc length of the curve

$$y = \frac{x^3}{3} + \frac{1}{4x}$$

from $x = 1$ to $x = 2$.

17. (a) Sketch the curve $y = x^3$ for $0 \le x \le 1$

(b) Sketch the surface obtained by rotating this curve about the x axis.

(c) Find the area of the surface in part (b).

18. A cylindrical tank with a radius of 2 m is half full of water. Find the hydrostatic force (in Newtons) on one end of the tank. Use the values $\rho = 10^3\,\mathrm{kg/m^3}$ for the density of water and $g = 9.8\,\mathrm{N/kg}$ for the acceleration of gravity.

19. Find the centroid of the triangular region with vertices at $(0,0)$, $(3,0)$ and $(0,4)$.

20. (a) Find a formula for the n-th term a_n (starting at $n = 1$) of the sequence

$$\frac{\sqrt{1\cdot 2}}{1}, \frac{\sqrt{2\cdot 3}}{3}, \frac{\sqrt{3\cdot 4}}{5}, \frac{\sqrt{4\cdot 5}}{7}, \ldots$$

(b) Find the limit $\lim_{n\to\infty} a_n$ of this sequence.

21. For each of the following, decide whether the sequence converges, diverges to ∞ or $-\infty$, or diverges without going to $\pm\infty$.

(a) $\lim_{n\to\infty} n\cos(\pi n)$

(b) $\lim_{n\to\infty} \cos(\pi n)/n$

(c) $\lim_{n\to\infty} n^3/2^n$

(d) $\lim_{n\to\infty} \sqrt{n} + \arctan(n^2)$

22. Express the repeating decimal $3.272727\ldots$ as a rational number p/q, where p and q are integers with no common factor.

23. Find the sum of the geometric series $2 - \frac{4}{3} + \frac{8}{9} - \frac{16}{27} + \cdots$.