Math 1B—Calculus, Spring 2018—Haiman
Practice problems for Final Exam

1. Evaluate the integral.

$$\int_0^2 e^{\sqrt{x}}\, dx$$

2. Evaluate the integral.

$$\int \sec^2(x)\tan^2(x)\, dx$$

3. Evaluate the integral.

$$\int_0^{\pi/2} \sin(x)\cos(2x)\, dx$$

4. Evaluate the integral.

$$\int \frac{1}{\sqrt{x^2-25}}\, dx$$

5. Evaluate the integral.

$$\int_0^1 \sqrt{x-x^2}\, dx$$

6. Evaluate the integral.

$$\int \frac{1}{(x^2+1)^{3/2}}\, dx$$

7. Evaluate the integral.

$$\int_0^1 \frac{x}{x^3+1}\, dx$$

8. Approximate the integral in Problem 5 using the midpoint rule, the trapezoidal rule, and Simpson's rule, using two subdivisions of the interval for the midpoint and trapezoidal rules, and four for Simpson's rule.

Which result gives the biggest overestimate, which gives the biggest underestimate, and which is most accurate? Explain why these occurred in the way that they did.

9. Determine whether the improper integral converges, and evaluate it if it does.

$$\int_{-\infty}^{\infty} \frac{1}{x^2+1}\, dx$$

10. Explain why the following integral is improper. Then determine whether it converges, and evaluate it if it does.

$$\int_0^{\pi} \tan(x)\, dx$$

11. Find the arc length of the curve $y = 4 - x^{3/2}$ from the point $(0,4)$ to the point $(4,-4)$.

12. The solid of revolution of an ellipse is called an *ellipsoid*. Find the surface area of an ellipsoid having a long axis of length 4, and whose cross section perpendicular to the midpoint of this axis is a circle of diameter 2.

Hint: the equation of an ellipse centered at the origin in the plane has the form $x^2/a + y^2/b = 1$.

13. Find the force due to hydrostatic pressure on a trapezoidal dam 10 meters high, 15 meters wide at the base, and 25 meters wide at the top, if the water behind it is 8 meters deep. Take the weight per unit volume of water to be 9800 kilograms per cubic meter.

14. Find the centroid of the region bounded by the x axis and the curve $y = \cos x$ from $x = -\pi/2$ to $x = \pi/2$.

15. Find the Taylor series expansion for the function $f(x) = 1/x$ about $x = 2$. What is its radius of convergence?

16. Find the first four non-zero terms of the Maclaurin series expansion of the function $f(x) = \sqrt{1 - x^2}$. Don't try to solve this problem by differentiating $f(x)$ repeatedly.

17. Determine whether the series is divergent, conditionally convergent, or absolutely convergent, and give a reason.

$$\sum_{n=1}^{\infty}(1 + \frac{1}{n^2})$$

18. Determine whether the series is divergent, conditionally convergent, or absolutely convergent, and give a reason.

$$\sum_{n=2}^{\infty} \frac{(-1)^n}{n(\ln n)^2}$$

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20. Determine whether the series is divergent, conditionally convergent, or absolutely convergent, and give a reason.

$$\sum_{n=0}^{\infty} \frac{(n!)^2}{(2n)!}$$

21. Find the radius of convergence of the Maclaurin series expansion of the function $f(x) = 1/(x^2 + 4)$.

22. Show that the error in approximating the infinite sum

$$1 + \frac{1}{4} + \frac{1}{9} + \frac{1}{16} + \frac{1}{25} + \dots$$

by the sum of its first five terms is at most $1/5$.

23. (a) Verify that the function defined by $f(x) = \frac{e^x - 1}{x}$ for $x \neq 0$, and $f(0) = 1$, is continuous.

(b) Find the Maclaurin series expansion of $f(x)$.

(c) Find the radius of convergence of this series.

24. Use Euler's method to approximate the solution of the initial value problem

$$y' = (x+y)^2, \quad y(0) = 1$$

at $x = 0$, $x = 0.1$, $x = 0.2$, and $x = 0.3$. You will need a calculator for this, but you will not need one for any question that might be on the exam.

25. Solve the initial value problem

$$y' = xy + x + y + 1, \quad y(0) = 0$$

by using the fact the the equation is separable.

26. Use the fact the equation in Problem 25 is linear to solve it another way.

27. Solve the initial value problem.

$$y'' + 4y' + 4y = 0, \quad y(0) = 1,\ y'(0) = -2$$

28. Find the general solution of the differential equation.

$$y'' + 5y' + 6y = e^{2x} + e^{-2x}$$

29. Solve the boundary value problem.

$$y'' + 2y' + 2y = 2, \quad y(0) = y(\pi/2) = 0.$$

30. A truck has a mass of 20,000 kg. The springs in its suspension exert a restoring force of 2×10^6 Newtons per meter of displacement from equilibrium.

(a) If the truck has no shock absorbers and hits a bump in the road, what is the period (in seconds) of the bouncing oscillation that results?

(b) If the truck is equipped with shock absorbers providing resistance of 2×10^5 N/(m/s), will it return to equilibrium without oscillating after hitting a bump, or will it still oscillate?

31. Find the first four non-zero terms in the Maclaurin series for the solution $y = f(x)$ of the initial value problem.

$$y'' + (x-1)y' + y = 0, \quad y(0) = 1,\ y'(0) = 0$$

32. (a) Show that the differential equation

$$xy' + y = e^x$$

has only one solution representable as a Maclaurin series $y = \sum_{n=0}^{\infty} c_n x^n$, and find this series.

(b) Find the general solution of the equation in part (a) by another method. Explain why solving for y as a Maclauin series did not give the full general solution.