1. Sub $u=\sqrt{x}$ and integrate by parts to get $2+2(\sqrt{2}-1)e^{\sqrt{2}}$

2. Save a factor $\sec^2 x$ and set $u=\tan x$ to get $\frac{1}{3}\tan^3 x + C$

3. Use
$$\sin 3x = \sin x \cos 2x + \sin 2x \cos x$$
$$\sin x = \sin(-x)\cos 2x + \sin 2x \cos(-x)$$
$$= -\sin x \cos 2x + \sin 2x \cos x$$

$\frac{1}{2}(\sin 3x - \sin x) = \sin x \cos 2x$ to get $-\frac{1}{6}\cos 3x + \frac{1}{2}\cos x + C$,
then $\int_0^{\pi/2} \cdots = -\frac{1}{3}$

4. use $x = 5\sec\theta$, get $\ln|x+\sqrt{x^2-25}| + C$

5. Complete the square: $x - x^2 = \frac{1}{4} - (x-\frac{1}{2})^2$, then use $x = \frac{1}{2}+\frac{1}{2}\sin\theta$, answer $\pi/8$. [or recognize the $\int$ as giving the area of a semicircle with radius $1/2$]

6. Use $x = \tan\theta$, get $\frac{x}{\sqrt{x^2+1}} + C$

7. Use partial fractions $\frac{x}{x^3+1} = \frac{1}{3}\frac{x+1}{x^2-x+1} - \frac{1}{3}\frac{1}{x+1}$, complete the square $x^2-x+1 = (x-\frac{1}{2})^2 + 3/4$, and integrate to get $\frac{\pi}{3\sqrt{3}} - \frac{\ln(2)}{3}$.

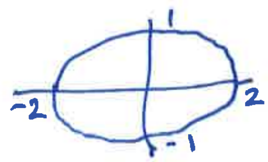
8. Midpoint $M_2 = \sqrt{3}/4$, Trapezoidal $T_2 = \frac{1}{4}$, Simpson $S_4 = \frac{1}{3}T_2 + \frac{2}{3}M_2$ $= \frac{2\sqrt{3}+1}{12}$. $M_2 \approx .433$ is over, $T_2 = .25$ is under, $S_4 \approx .372$ is the closest to $\pi/8 \approx .392\ldots$. The graph of $\sqrt{x-x^2}$ is concave downward; this is why T_2 is under and M_2 is over. In S_4, these errors tend to cancel.

9. $\int \frac{1}{x^2+1}\,dx = \tan^{-1}(x)$ has $\lim_{x\to\infty}\tan^{-1}(x) = \pi/2$, $\lim_{x\to-\infty}\tan^{-1}(x) = -\pi/2$,
so $\int_{-\infty}^{\infty}\frac{1}{x^2+1}\,dx$ converges to $\tan^{-1}(x)\Big]_{-\infty}^{\infty} = \pi$.

10. The integral is improper because $\tan x$ has a discontinuity at $\pi/2$. To see if it converges we need to evaluate
$\lim_{a\to\pi/2^-}\int_0^a \tan x\,dx$ and $\lim_{a\to\pi/2^+}\int_a^{\pi}\tan x\,dx$. Since
$\int \tan x\,dx = \ln|\sec x| \to -\infty$ as $x \to \pi/2$, the limits don't exist, and the integral diverges.

11. Find $ds = \sqrt{(y')^2+1}\,dx = \sqrt{\frac{9x}{4}+1}$, then find $\int_0^4 \sqrt{\frac{9x}{4}+1}\,dx =$
$\frac{8}{27}(10\sqrt{10}-1)$.

12. The ellipse is $\frac{x^2}{4}+y^2=1$.

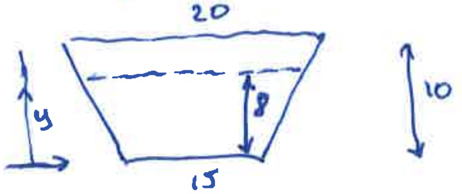


For $y=\sqrt{1-x^2/4}$, the area element

$dA = 2\pi y\, ds$ is $\frac{\pi}{2}\sqrt{16-3x^2}$.

Find $\int_{-2}^{2} \frac{\pi}{2}\sqrt{16-3x^2}\,dx = 2\pi + \frac{8\pi^2}{3\sqrt{3}}$ using trig sub $x=\frac{4}{\sqrt{3}}\sin\theta$, or integral table.

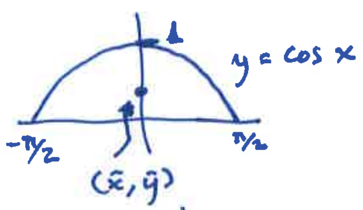
13.



Taking coordinate $y=0$ at bottom of dam, the width of the dam at height y is $15+y/2$, and the water depth is $8-y$, giving force $\int_0^8 \rho g(8-y)(15+y/2)\,dy$

$= \frac{1568}{3}\rho g = \frac{1568\cdot 9800}{3}$ N

14.



By symmetry, $\bar{x}=0$.
The area is $A=\int_{-\pi/2}^{\pi/2}\cos x\,dx = \sin x\Big]_{-\pi/2}^{\pi/2} = 2$.

To find $\bar{y}$, can either use

$\bar{y} = \frac{1}{A}\int_0^1 y\cdot 2\cos^{-1}(y)\,dy$ or $\bar{y} = \frac{1}{A}\int_{-\pi/2}^{\pi/2}\frac{\cos^2 x}{2}\,dx$, getting $\bar{y} = \pi/8$. (For the first method, integrate by parts; for the 2nd, use half-angle formula $\cos^2 x = \frac{1+\cos 2x}{2}$).

15. $\frac{1}{x} = \frac{1}{2-(2-x)} = \frac{1/2}{1-(-1/2)(x-2)} = \frac{1}{2}\sum_{n=0}^{\infty}\left(-\frac{1}{2}\right)^n (x-2)^n = \sum_{n=0}^{\infty}\frac{(-1)^n}{2^{n+1}}(x-2)^n$,

by geometric series. The radius of convergence is 2.

16. Substitute $-x^2$ for x in the binomial series $(1+x)^{1/2} = \sum_{n=0}^{\infty}\binom{1/2}{n}x^n$

to get $1-\frac{x^2}{2}-\frac{x^4}{8}-\frac{x^6}{16}+\cdots$

17. Divergent because $1+\frac{1}{n^2}$ does not go to 0 as $n\to\infty$.

18. The absolute value series

$$\sum_{n=2}^{\infty} \frac{1}{n(\ln n)^2}$$

converges by the integral test: $\int_2^\infty \frac{1}{x(\ln x)^2}\,dx = \int_{\ln 2}^\infty \frac{1}{u^2}\,du$ ($u=\ln x$) converges. So the series is absolutely convergent. The alternating series test also shows that is convergent, but absolutely convergent is stronger.

19. Convergent by alt. series test, but not absolutely convergent, by $\int$ test applied to $\sum_{n=2}^{\infty} \frac{1}{n \ln n}$. So, conditionally convergent.

20. Use the ratio test:

$$\lim_{n\to\infty} \frac{(n+1)!^2}{(2n+2)!} \frac{(2n)!}{(n!)^2} = \lim_{n\to\infty} \frac{(n+1)^2}{(2n+2)(2n+1)} = \frac{1}{4} < 1 \Rightarrow \text{convergent.}$$

The series is positive, so it's absolutely convergent.

21. It's a geometric series $\frac{1}{x^2+4} = \frac{1/4}{1+(x^2/4)}$, convergent for $|x^2/4|<1$, $|x|<2$, so the radius of convergence is 2.

22. By the $\int$ test, the remainder term $R_5 = \frac{1}{36}+\frac{1}{49}+\cdots$ is less than $\int_5^\infty \frac{1}{x^2}\,dx = -\frac{1}{x}\Big]_5^\infty = \frac{1}{5}$.

23. a) $f(x)$ is continuous because $\lim_{x\to 0} \frac{e^x-1}{x} = 1$ (by L'Hospital, or by definition of $y'(0)$ for $y=e^x$).

b) $e^x - 1 = x + \frac{x^2}{2!} + \frac{x^3}{3!} + \cdots$

Divide by x to get $f(x) = 1 + \frac{x}{2!} + \frac{x^2}{3!} + \cdots = \sum_{n=0} \frac{x^n}{(n+1)!}$

c) The radius of convergence is ∞ because the series for e^x converges for all x.

24.

x	0	.1	.2	.3
y	1	1.1	1.244	1.45251

25. Factor: $y' = (x+1)(y+1)$

$\frac{dy}{y+1} = x+1\, dx \xrightarrow{\int} \ln|y+1| = \frac{x^2}{2} + x + c$

$y = Ae^{x^2/2 + x} - 1$. $y(0) = 0 \Rightarrow A = 1$,

$y = e^{x^2/2 + x} - 1$.

26. $y' - (x+1)y = x+1$. Integrating factor $e^{\int -(x+1)\,dx} = e^{-(\frac{x^2}{2}+x)}$

gives $\left(e^{-(\frac{x^2}{2}+x)} y\right)' = e^{-(\frac{x^2}{2}+x)}(x+1)$.

Integrate both sides, using $u = \frac{x^2}{2} + x$, $du = (x+1)dx$ on the right:

$e^{-(\frac{x^2}{2}+x)} y = -e^{-(\frac{x^2}{2}+x)} + C$.

$y = Ce^{\frac{x^2}{2}+x} - 1$, $y(0) = 0 \Rightarrow C = 1$, $y = e^{\frac{x^2}{2}+x} - 1$.

27. $r^2 + 4r + 4 = (r+2)^2 = 0$ has double root $r = -2$, so

$y = (A + Bx)e^{-2x}$. $y(0) = A = 1$

$y'(0) = B - 2A = -2 \Rightarrow B = 0$

$y = e^{-2x}$.

28. For homogeneous eq'n $y'' + 5y' + 6y = 0$, $r^2 + 5r + 6 = (r+2)(r+3) = 0$

has roots $r = -2, -3$, giving $y_c = C_1 e^{-2x} + C_2 e^{-3x}$.

Since e^{-2x} is a solution of the complementary equation, find y_p

using $y_p = Ae^{2x} + Bxe^{-2x}$

$y_p' = 2Ae^{2x} + B(1-2x)e^{-2x}$

$y_p'' = 4Ae^{2x} + 4B(x-1)e^{-2x}$

$y_p'' + 5y_p' + 6y_p = 20Ae^{2x} + Be^{-2x} = e^{2x} + e^{-2x}$

$\Rightarrow A = \frac{1}{20}$, $B = 1$,

$y = y_p + y_c = \frac{1}{20}e^{2x} + xe^{-2x} + C_1 e^{-2x} + C_2 e^{-3x}$

29. For the homog. eq'n $y^2+2y'+2=0$, $r = \frac{-2 \pm \sqrt{4-8}}{2} = -1 \pm i$,

So $y_c = e^{-x}(A\cos x + B\sin x)$. Take $y_p = C$ then

$y_p'' + 2y_p' + 2y_p = 2C = 2 \Rightarrow C=1$, so $y = y_p + y_c = 1 + e^{-x}(A\cos x + B\sin x)$.

$y(0) = 1+A = 0 \quad \Rightarrow A = -1$

$y(\pi/2) = 1 + e^{-\pi/2}B = 0 \quad \Rightarrow B = -e^{\pi/2}$

$y = 1 - e^{-x}(\cos x + e^{\pi/2}\sin x)$.

30. a) Period of oscillation is $2\pi\sqrt{\frac{m}{k}} = \frac{2\pi}{10} \approx .628$ sec

b) With $m = 2\cdot 10^4$, $c = 2\cdot 10^5$, $k = 2\cdot 10^6$,

$my'' + cy' + ky = 0$ has $c^2 - 4km < 0$, so the solution has exponentially decaying oscillations.

31. Plug in $y = \sum_{n=0}^{\infty} c_n x^n$, $y' = \sum_{n=0}^{\infty}(n+1)c_{n+1}x^n$, $xy' = \sum_{n=0}^{\infty} nc_n x^n$,

$y'' = \sum_{n=0}^{\infty}(n+2)(n+1)c_{n+2}x^n$ to get

$$\sum_{n=0}^{\infty}\Big(\underbrace{(n+2)(n+1)c_{n+2} + nc_n - (n+1)c_{n+1} + c_n}_{=(n+1)((n+2)c_{n+2} - c_{n+1} + c_n)}\Big)x^n = 0$$

So $c_{n+2} = \frac{c_{n+1} - c_n}{n+2}$ for all $n = 0, 1, \dots$

Initial conditions give $c_0 = y(0) = 1$, $c_1 = y'(0) = 0$.

Then $(n=0)$ $c_2 = \frac{c_1 - c_0}{2} = \frac{-1}{2}$

$(n=1)$ $c_3 = \frac{c_2 - c_1}{3} = -\frac{1}{6}$

$(n=2)$ $c_4 = \frac{c_3 - c_2}{4} = \frac{1}{12}$

$y = 1 - \frac{1}{2}x^2 - \frac{1}{6}x^3 + \frac{1}{12}x^4 + \dots$

32. a) Plug in $y = \sum_{n=0}^{\infty} c_n x^n$ $\quad xy' = \sum_{n=0}^{\infty} n c_n x^n$:

$$\sum_{n=0}^{\infty} (n+1) c_n x^n = \sum_{n=0}^{\infty} \frac{x^n}{n!}$$

Equate coefficients : $(n+1) c_n = \frac{1}{n!}$ $\quad c_n = \frac{1}{(n+1)!}$

$y = 1 + \frac{x}{2!} + \frac{x^2}{3!} + \dots = \sum_{n=0}^{\infty} \frac{x^n}{(n+1)!}$ is the only series solution

(this is the same function as in problem 23).

b) The equation $y' + \frac{y}{x} = e^x/x$ is linear, with integrating factor $e^{\int \frac{1}{x} dx} = x$, giving back the original equation

$$\underbrace{xy' + y}_{(xy)'} = e^x$$

Integrate both sides to get

$$xy = e^x + C$$

$$y = \frac{e^x + C}{x} .$$

This general solution has an undetermined constant, as expected. But it is discontinuous at $x=0$ unless $C=-1$, in which case $f(x) = \frac{e^x - 1}{x}$ extends to a continuous function with $f(0) = 1$.

This $f(x)$ is the only possible Maclaurin series solution, since such a solution can't be discontinuous at $x=0$.