Name Solutions   Student ID____________

Section time & instructor ____________

Math 1A—Calculus, Spring 2017—Haiman
Midterm Exam 2

Instructions:

- Wait until time to start is announced before looking at the exam questions.

- Write your name, ID number and section time and instructor's name on this page now. Before turning in your exam, write your name and ID number where indicated on the first side of the second page.

- Write your answers on the exam paper in the space provided. Do preliminary work on scratch paper, then write a clear and concise answer giving your solution and enough steps to justify it.

  Even if you have a correct answer, points may be deducted if parts of what you write are incorrect or irrelevant.

  If you need more space for an answer, attach an extra page, and write your name and ID number on it. Ordinarily, extra pages should not be needed.

- You may use one sheet (written on both sides) of prepared notes. No other notes, books, calculators, or other electronic devices are allowed.

- There are 6 questions, for a total of 100 points.

1. (16 pts) Evaluate the limit:

$$\lim_{x\to\infty} \frac{\sqrt{3x^6 - 2x^3 + 5}}{7 - x^3}$$

Divide numerator and denominator by x^3 to get

$$\lim_{x\to\infty} \frac{\sqrt{3 - 2x^{-3} + 5x^{-6}}}{7x^{-3} - 1} = -\sqrt{3},$$

since all negative powers of x go to zero.

2. (16 pts) Find the derivative of $\sin(x + \sin \pi x)$.

$$\cos(x + \sin \pi x)(1 + \pi \cos \pi x)$$

Name ______________________ Student ID______________________

3. (16 pts) Find the derivative of $x^{(x^2)}$.

Use logarithmic differentiation: $f'(x) = f(x)\,(\ln f(x))'$, to get

$$x^{(x^2)}\,(x^2 \ln x)' = x^{(x^2)}\,(2x \ln x + x)$$

4. (18 pts) Find an equation of the tangent line to the curve

$$xy + \ln(\frac{x+y}{3}) = 2$$

at the point $(2, 1)$.

Differentiate implicitly to get

$$y + xy' + \frac{3}{x+y}\,\frac{1+y'}{3} = 0$$

$$(x + \frac{1}{x+y})\,y' = -y - \frac{1}{x+y}.$$

At $(2,1)$, this gives

$$\frac{7}{3}y' = -\frac{4}{3}$$

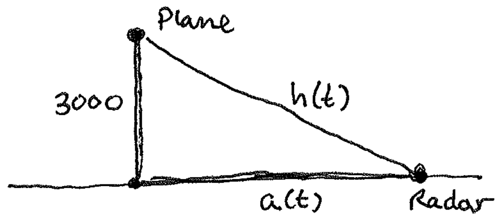
$$y' = -4/7.$$

The tangent line is $y - 1 = -\frac{4}{7}(x-2)$, or $y = \frac{15-4x}{7}$

5. (18 pts) An airplane in level flight at an altitude of $3000\,m$ is approaching a radar station on the ground straight ahead. The radar station detects that the distance to the plane is $5000\,m$, and that this distance is decreasing at a rate of $200\,m/s$. At what speed is the plane flying?

The radar measures distance in a straight line from its antenna on the ground to the plane in the air.

Consider the right triangle formed by the plane, the radar, and the point on the ground below the plane:



We are given that the vertical side is $3000\,m$. We are to find $-a'$ when $h = 5000$ and $h' = -200$.

Differentiating on both sides in $h^2 = a^2 + 3000^2$ gives

$2hh' = 2aa'$, $a' = \dfrac{hh'}{a}$. When $h = 5000$, $a = \sqrt{5000^2 - 3000^2} = 4000$.

Then $a' = \dfrac{5000\,(-200)}{4000} = -250$, $-a' = \boxed{250\ m/s}$

6. (16 pts) Use a linear approximation or differentials to estimate the value of $\sqrt[3]{10}$, based on the known exact value $\sqrt[3]{8} = 2$.

To find the linear approximation to $y = x^{1/3}$ at $a = 8$, compute $y(a) = 2$, and $y' = \frac{1}{3}x^{-2/3}$, $y'(a) = \dfrac{1}{3 \cdot 8^{2/3}} = \dfrac{1}{12}$,

so the approximation is

$$y \approx \frac{1}{12}(x - 8) + 2.$$

At $x = 10$, this gives

$$\sqrt[3]{10} \approx \frac{1}{6} + 2 = \frac{13}{6}.$$

Using differentials, $dy = y'(a)\,dx = \dfrac{dx}{12}$ gives $dy = \frac{1}{6}$ when $dx = 2$, so $\sqrt[3]{10} \approx y(a) + dy = 2 + \frac{1}{6} = \frac{13}{6}$.

By the way, $\sqrt[3]{10} = 2.154\ldots$, so $13/6 = 2.166\ldots$ is a pretty good approximation.