Name _____ Section time & instructor _____

Student ID_____

Math 1A—Calculus, Spring 2014—Haiman
Final Exam

Instructions:

• Write your name, ID number, discussion section instructor's name, and discussion section time at the top of this page. Do not look at the other pages until the signal to start is given.

• You may use two sheets (written on both sides) of prepared notes. No other notes, books, calculators, or other electronic devices are allowed.

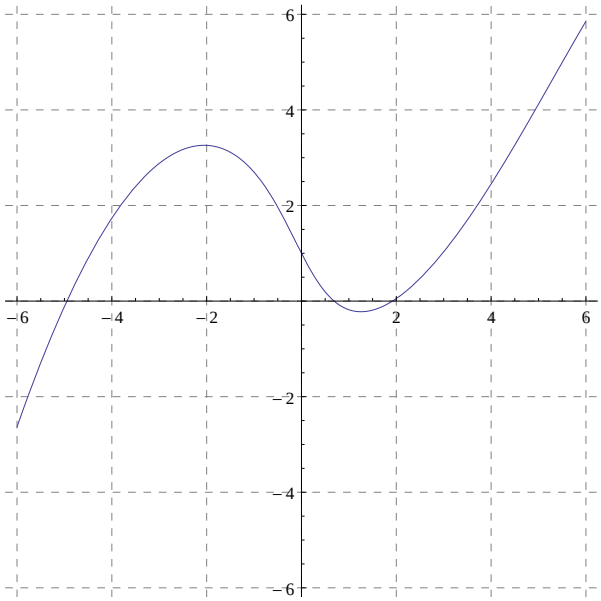
• Use scratch paper for preliminary work, then write your solutions on the exam paper. Hand in **only the exam paper** itself.

• Show enough steps to indicate how you got your answer. An answer that is just a number or formula without explanation will receive no credit if wrong, and might not receive full credit even if correct.

• There are 16 questions, 100 total points.

For grading use only			
1		9	
2		10	
3		11	
4		12	
5		13	
6		14	
7		15	
8		16	
Total:			

1. (6 pts) The graph of a function $f(x)$ is shown. On the same grid, sketch the graphs of $g(x) = 2f(x) - 3$ and $h(x) = f(2x - 3)$. Label your two graphs so we can tell which is which.



2. (6 pts) Evaluate the limit if it exists, either as a number or an infinite limit.

$$\lim_{x \to \infty} 5\sqrt{x} - 3x$$

3. (6 pts) Evaluate the limit if it exists, either as a number or an infinite limit.

$$\lim_{x \to \pi/2} \ln(2x/\pi) \sec x$$

4. (6 pts) Find the equation of the tangent line to the curve $x^3 + y^3 = 9(x - y)$ at the point $(2, 1)$.

5. (6 pts) Find the maximum and minimum values of the function $f(x) = x^4 - 8x^2$ on the interval $[-1, 3]$.

6. (6 pts) Differentiate the function $f(x)$ defined by

$$f(x) = \int_0^{x^2} \sec(u)\, du.$$

You do not need to evaluate the integral to solve this problem.

7. (a) (4 pts) Show that Newton's method for computing a sequence of approximations $x_1, x_2, \ldots$ to $\sqrt{a}$ leads to the formula

$$x_{n+1} = \frac{x_n + a/x_n}{2}$$

(b) (2 pts) Find the second and third approximations to $\sqrt{2}$ given by this scheme when the first approximation is $x_1 = 1$.

8. (6 pts) Find the first and second derivatives $f'(x)$ and $f''(x)$ of the function

$$f(x) = \cos(\ln x)$$

9. (6 pts) If you know that $-1 \leq f'(x) \leq 1$ on the interval $[0, 2]$, and $f(0) = 8$, what can you conclude about the value of $f(2)$, and why?

10. (7 pts) For the function $f(x) = 1/x^2$, find the point c in the interval $[1, 4]$ where $f(c)$ is equal to the average value of $f(x)$ on $[1, 4]$.

11. (6 pts) An airplane is flying horizontally, due north, at an altitude of 3 miles and a speed of 500 miles per hour. A radar station is on the ground ahead. At what rate is the distance between the plane and the radar station decreasing when the plane is 4 miles south of the station?

12. (6 pts) Evaluate the indefinite integral

$$\int \frac{x^2}{\sqrt{3x^3+2}}\,dx$$

13. (6 pts) Evaluate $\int_0^5 f(x)\,dx$, where

$$f(x) = \begin{cases} x^2 & x < 1 \\ 1/x & x \geq 1 \end{cases}$$

14. (7 pts) Evaluate the definite integral

$$\int_0^{\sqrt{2}} \sqrt{4 - x^2} - x \, dx$$

by interpreting it as an area.

15. (7 pts) Find the area of the region enclosed between the parabolas $y = x^2 - x - 1$ and $y = -x^2 - x + 1$.

16. (7 pts) Find the volume of the solid obtained by rotating the triangle with vertices $(0, 1)$, $(0, -1)$ and $(3, 0)$ about the y axis.