Name ______________________ Section time & instructor ______________________

Student ID______________________

Math 1A—Calculus, Spring 2014—Haiman
Midterm Exam 1

Instructions:

• Write your name, ID number and discussion section time and instructor's name at the top of this page. Do not look at the other pages until the signal to start is given.

• You may use one sheet (written on both sides) of prepared notes. No other notes, books, calculators, or other electronic devices are allowed.

• Use scratch paper for preliminary work, then write your solutions on the exam paper. Hand in **only the exam paper** itself.

• Show enough steps to indicate how you got your answer. An answer that is just a number or formula without explanation will receive no credit if wrong, and might not receive full credit even if correct.

• There are 8 questions, for a total of 100 points.

1. (8 pts each part) (a) Find the domain of the function $f(x) = \sqrt{\log_2 x}$.

Need x in domain of $\log_2 x$, so $x \geq 0$, <u>and</u> $\log_2 x$ in domain of $\sqrt{x}$, so $\log_2 x \geq 0$. This second condition means $x \geq 1$. So the domain of $\sqrt{\log_2 x}$ is

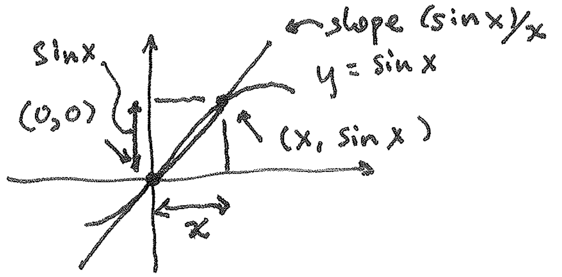
$$[1, \infty).$$

(b) If $f(x)$ is one-to-one, find its inverse function. If $f(x)$ is not one-to-one, explain why.

It is one-to-one, and the inverse function is

$$f^{-1}(x) = 2^{(x^2)}.$$

2. (12 pts) Express the slope of the tangent line to the graph $y = \sin x$ at the point $(0, 0)$ as a limit. You do not have to evaluate the limit.



$$\lim_{x \to 0} \frac{\sin x}{x}$$

3. (12 pts) Which one of the intervals

$$(0, 1), \quad (1, 4) \quad \text{or} \quad (4, 9)$$

contains a root of the equation

$$\sqrt{x} = \frac{x-1}{10-x},$$

and why? You may take as known that only one of these intervals contains a root.

x	$\sqrt{x}$	$\frac{x-1}{10-x}$	$\sqrt{x} - \frac{x-1}{10-x}$
0	0	$-1/10$	$1/10$
1	1	0	1
4	2	$1/2$	$3/2$
9	3	8	-5

Since $\sqrt{x} - \frac{x-1}{10-x}$ is continuous, positive at $x=4$, and negative at $x=9$, it is zero somewhere in the interval $\boxed{(4,9)}$, by I.V.T..

4. (12 pts) Evaluate the limit (possibly as an infinite limit) or explain why it does not exist.

$$\lim_{x\to-\infty} \cos(3^x + \pi)$$

Since $3>1$, 3^x is small for x large and negative, i.e. $\lim_{x\to-\infty} 3^x = 0$. By limit laws and the fact that $\cos x$ is continuous, it follows that

$$\lim_{x\to-\infty} \cos(3^x+\pi) = \cos(0+\pi) = \cos(\pi) = -1 .$$

5. (12 pts) Evaluate the limit (possibly as an infinite limit) or explain why it does not exist.

$$\lim_{x\to\pi/2} \frac{1}{1-\cancel{\cos}\,\sin x}$$

Since $\cancel{\cos}\,\sin\frac{\pi}{2} = 1$ and $\cancel{\cos}\,\sin x$ is continuous, $\cancel{\cos}\,\sin x \to 1$ as $x\to\pi/2$. Since $\cancel{\cos}\,\sin x \le 1$ for all x, $1-\cancel{\cos}\,\sin x$ is small and positive for x near $\pi/2$. Therefore,

$$\lim_{x\to\pi/2} \frac{1}{1-\cancel{\cos}\,\sin x} = \infty$$

6. (12 pts) Find all horizontal and vertical asymptotes of the graph

$$y = \frac{2x^2-5x+9}{x^2-9}.$$

The function $f(x) = \frac{2x^2-5x+9}{x^2-9}$ has infinite limits at $x\to3^+$, $x\to3^-$, $x\to(-3)^+$, $x\to(-3)^-$, and $\lim_{x\to\pm\infty} f(x) = 2$. Accordingly, the lines $x=3$ and $x=-3$ are ~~horizontal~~ vertical asymptotes, and $y=2$ is a horizontal asymptote.

7. (12 pts) Find the constant C that makes the function

$$f(x) = \begin{cases} 2x + C & x \leq 2 \\ \dfrac{x^2 - 3x + 2}{x - 2} & x > 2 \end{cases}$$

continuous on $(-\infty, \infty)$.

We have $\lim_{x \to 2^+} f(x) = \lim_{x \to 2^+} \frac{(x-1)(x-2)}{x-2} = \lim_{x \to 2^+} (x-1) = 1.$

We need $\lim_{x \to 2^-} f(x) = 1$, so $4 + C = 1$, $C = -3$.

8. (12 pts) Fill in the missing numbers, functions or symbols indicated by the lettered boxes in the definition of

$$\lim_{x \to 3} \frac{1}{x+4} = 1/7.$$

For every number $\epsilon > 0$ there is a number $\delta > 0$ such that

if $0 < |x - \boxed{A}| < \boxed{B}$ then $|\boxed{C} - 1/7| < \boxed{D}$.

$\boxed{A}$ 3

$\boxed{B}$ δ

$\boxed{C}$ $\frac{1}{x+4}$

$\boxed{D}$ ϵ