

Final Examination

Name _____

Student ID _____

Discussion Section (Time and GSI's name) _____

Instructions:

- Do not look at the exam questions before the start of the exam is announced.
- Write your name on each page in case they get separated.
- Write answers in the space provided and turn in only the exam paper. Show enough work so that we can see how you arrived at your answers.
- You may use one sheet of notes. No other notes, books or calculators allowed.
- The exam has 3 pages (both sides) and 20 questions. All questions have equal value.

<i>Grading use only</i>			
1		11	
2		12	
3		13	
4		14	
5		15	
6		16	
7		17	
8		18	
9		19	
10		20	
Total:			

1. Simplify $x^{1/\ln x}$.

2. If $f(x)$ is continuous on $[0, 2]$, and $f(0) = 1$, $f(1) = 2$, $f(2) = 0$, show that f is not one-to-one.

3. Find the equation of the tangent line to $x^3 + y^3 = 9$ at $(2, 1)$.

4. Evaluate the limit (as a number or an infinite limit):

$$\lim_{x \rightarrow \pi/2} \frac{1 - \sin x}{\cos^2 x}$$

5. Evaluate the limit (as a number or an infinite limit):

$$\lim_{x \rightarrow +\infty} (1 + 2/x)^x$$

6. Find c such that the line $y = x + c$ is a slant asymptote to the curve $y = x^2/(x + 5)$.

7. Find $(d/dx)^{17}(e^x + e^{-x})$.

8. If X and Y are functions of t related by $Y = e^{XY}$, find X' when $Y = 1$ and $Y' = 3$.

9. Find the point on the line $x + 2y = 3$ closest to the origin.

10. Find all local minima and maxima of the function $f(x) = x^2 e^{-x}$, and the intervals where f is increasing or decreasing.

11. Show that the equation $x^3 - 3x + 3 = 0$ has exactly one real root.

12. Using Newton's method to find an approximate solution to the equation $x^3 = 2$, starting with first approximation $x_1 = 1$, find the next approximation.

13. Find $f(x)$ such that $f''(x) = 1 + \sin x$, $f(0) = 0$, and $f'(0) = 0$.

14. Show that $\int_0^1 e^{-x^2} dx \leq (1 + e^{-1/4})/2$.

15. Differentiate the function $F(x) = \int_1^{1/x} \sin^{-1}(t) dt$

16. Evaluate the integral $\int_{-1}^2 |x^3| dx$.

17. Evaluate the integral $\int_1^e \sqrt{\ln x}/x \, dx$.

18. Find the area of the region enclosed by the line $x = 1$ and the curves $8y = x^2$ and $xy = 1$.

19. Find the volume of the solid of rotation about the y -axis of the region in the first quadrant enclosed by the y -axis, the line $y = x + 1$, and the curve $y = 2x^2$.

20. For the function $f(x) = 1/x$, find the point c in the interval $(1, 3)$ such that $f(c)$ is equal to the average value of f on the interval $[1, 3]$.