Name ______________________ Student ID ______________________

Section time & instructor ______________________

Math 1A—Calculus, Spring 2017—Haiman
Final Exam

Instructions:

• Wait until time to start is announced before looking at the exam questions.

• Write your name, ID number and section time and instructor's name on this page now. Before turning in your exam, write your name and ID number where indicated on the first side of each remaining page.

• Write your answers on the exam paper in the space provided. Do preliminary work on scratch paper, then write a clear and concise answer giving your solution and enough steps to justify it.

Points may be deducted for incorrect or irrelevant parts of a solution even if a correct answer is included.

If you need more space for your solution to a problem, attach an extra page and write your name and ID number on it. Extra pages should not normally be necessary.

• You may use two sheets (written on both sides) of prepared notes. No other notes, books, calculators, or other electronic devices are allowed.

• There are 18 questions, for a total of 100 points.

1. (a) (5 pts) Find the value of the constant A that makes the function

$$f(x) = \begin{cases} x^2 + Ax & x < 2 \\ \sqrt{x} & x \geq 2 \end{cases}$$

continuous for all x.

2. (5 pts) Simplify $x^{2/\ln x}$

3. (6 pts) For what values of x is the function $f(x) = |x+1| - |x-2|$ differentiable? Sketch the graph of the function and explain how your answer is related to the graph.

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4. (5 pts) Evaluate the limit, possibly as an infinite limit, or explain why it does not exist.

$$\lim_{x \to 0} \frac{x - \ln(x+1)}{1 - \cos(x)}$$

5. (5 pts) Evaluate the limit, possibly as an infinite limit, or explain why it does not exist.

$$\lim_{x \to \pi} \frac{\sin x}{x}$$

6. (5 pts) Evaluate the limit, possibly as an infinite limit, or explain why it does not exist.

$$\lim_{x \to \infty} (\ln(x) - x)$$

7. (6 pts) Differentiate the function $f(x) = \sqrt{1 + \sqrt{1 + \sqrt{x}}}$.

8. (6 pts) Find the maximum and minimum values of the function $f(x) = x + 4/x$ on the interval $[1, 3]$.

9. (6 pts) If $y \ln(y) = x$, find $y'(x)$ and $y''(x)$ at the point $(0, 1)$.

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10. (6 pts) The pressure P, volume V, and temperature T of a gas are related by $PV = 20T$. All three quantities vary with time. Find T and T' when $P = 40$, $V = 25$, $P' = 5$, and $V' = -2$.

11. (a) (3 pts) Without using any calculations, give a reason why there must be a point (a, b) on the graph $y = x^3$, with $0 < a < 1$, where the tangent line to the graph is parallel to its secant line through the points $(0, 0)$ and $(1, 1)$.

(b) (3 pts) Calculate the coordinates of the point (a, b) in the first part of the question.

12. (6 pts) A particle moves along the x-axis with velocity $v(t) = t^2 - 1$. It starts at time $t = 0$ at the origin, $x(0) = 0$. Find its position $x(2)$ at time $t = 2$, and the total distance travelled during the time interval $[0, 2]$. Note: the distance travelled is not necessarily the same as the final position.

13. (6 pts) By using Riemann sums with three terms to approximate the integral $\int_1^2 \frac{1}{x} dx$, find numbers B and C such that $B \leq \ln(2) \leq C$. The numbers B and C should be expressed as fractions with integer numerator and denominator.

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14. (5 pts) Evaluate the definite integral

$$\int_1^e \frac{\ln(x)}{x}\,dx$$

15. (5 pts) Evaluate the indefinite integral

$$\int x\sqrt{x+2}\,dx$$

16. (6 pts) Find the area of the region bounded by the parabola $y = x^2$ and the line $y = x+2$.

17. (a) (2 pts) Draw an accurate sketch of a region in the plane such that the solid of revolution obtained by rotating this region about the y axis is a cone with height 4 and circular base of radius 2.

(b) (4 pts) Set up and evaluate an integral which gives the volume of this cone, using your choice of method.

18. (5 pts) Find the number a such that the average value of the function $f(x) = x^2$ on the interval $[-a, a]$ is equal to 1.