Name _____ Section time & instructor _____

Student ID_____

Math 1A—Calculus, Spring 2014—Haiman
Midterm Exam 3

Instructions:

• Write your name, ID number and discussion section time and instructor's name at the top of this page. Do not look at the other pages until the signal to start is given.

• You may use one sheet (written on both sides) of prepared notes. No other notes, books, calculators, or other electronic devices are allowed.

• Use scratch paper for preliminary work, then write your solutions on the exam paper. Hand in **only the exam paper** itself.

• Show enough steps to indicate how you got your answer. An answer that is just a number or formula without explanation will receive no credit if wrong, and might not receive full credit even if correct.

• There are 6 questions, for a total of 100 points.

1. (a) (10 pts) Find all values of x which are critical numbers for the function

$$f(x) = \arctan(2x) - |x+1|$$

(b) (10 pts) Find all local and absolute minima and maxima of $f(x)$. Give the value of x and $f(x)$ for each one, and say what kind it is.

You may find the table below of values of $\arctan(x)$ helpful.

$$\begin{aligned} \arctan(-2) &\approx -1.107 \\ \arctan(-1) = -\pi/4 &\approx -0.785 \\ \arctan(0) &= 0 \\ \arctan(1) = \pi/4 &\approx 0.785 \\ \arctan(2) &\approx 1.107 \end{aligned}$$

2. (10 pts) Find the limit

$$\lim_{x \to 0} \frac{\ln(\cos x)}{x^2}.$$

3. (a) (7 pts) Show that the equation

$$4x^3 + 2 = 5x^2 + 2x$$

has a solution in the interval $[0, 1]$.

(b) (8 pts) Suppose you use Newton's method to try to find the solution, taking $x_1 = 0$ as your first approximation. What do you get for the second and third approximations?

(c) (5 pts) Assuming you found that something goes wrong in part (b), what strategy would you suggest to fix it? Answer in one sentence without making any further calculations.

4. (15 pts) Among all triangles with two sides lying along the x and y axes, and the third side passing through the point $(2, 3)$, find the one which has either the largest or the smallest area, and decide whether its area is in fact largest, or smallest.

5. (15 pts) Suppose you know the following information about a function $f(x)$:
   (a) $f(x)$ is an odd function, *i.e.*, $f(-x) = -f(x)$.
   (b) $f(x)$ is continuous for all x except -1 and 1, and $\lim_{x\to 1} f(x) = +\infty$.
   (c) $f'(x)$ is positive on $(-1, 1)$ and negative on $(1, \infty)$.
   (d) $f''(x)$ is positive on $(0, 1)$ and $(1, \infty)$.
   (e) $\lim_{x\to\infty} f(x) = 1$.

Sketch the graph of $f(x)$. Your graph should accurately reflect all the available information.

6. (a) (10 pts) Evaluate the integral

$$\int_0^1 \frac{4}{x^2+1}\, dx$$

(b) (10 pts) Use Riemann sum approximations to the integral in part (a), subdividing the interval $[0, 1]$ into two equal parts, to find rational numbers (*i.e.*, fractions with integer numerator and denominator) A and B such that

$$A < \pi < B.$$