1. (8 pts each part) (a) Find the domain of the function $f(x) = \sqrt{\log_2 x}$.

Need $x$ in domain of $\log_2 x$, so $x \geq 0$, and $\log_2 x$ in domain of $\sqrt{x}$, so $\log_2 x \geq 0$. This second condition means $x \geq 1$. So the domain of $\sqrt{\log_2 x}$ is $[1, \infty)$.

(b) If $f(x)$ is one-to-one, find its inverse function. If $f(x)$ is not one-to-one, explain why.

It is one-to-one, and the inverse function is $f^{-1}(x) = 2^{(x^2)}$.

2. (12 pts) Express the slope of the tangent line to the graph $y = \sin x$ at the point $(0, 0)$ as a limit. You do not have to evaluate the limit.

\[
\lim_{x \to 0} \frac{\sin x}{x}
\]

3. (12 pts) Which one of the intervals $(0, 1)$, $(1, 4)$ or $(4, 9)$ contains a root of the equation

$\sqrt{x} = \frac{x - 1}{10 - x}$

and why? You may take as known that only one of these intervals contains a root.

<table>
<thead>
<tr>
<th>$x$</th>
<th>$\sqrt{x}$</th>
<th>$\frac{x-1}{10-x}$</th>
<th>$\sqrt{x} - \frac{x-1}{10-x}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>$-\frac{1}{10}$</td>
<td>$\frac{1}{10}$</td>
</tr>
<tr>
<td>1</td>
<td>1</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>4</td>
<td>2</td>
<td>$\frac{1}{2}$</td>
<td>$3/2$</td>
</tr>
<tr>
<td>9</td>
<td>3</td>
<td>8</td>
<td>-5</td>
</tr>
</tbody>
</table>

Since $\sqrt{x} - \frac{x-1}{10-x}$ is continuous, positive at $x=4$, and negative at $x=3$, it is zero somewhere in the interval $(4,9)$, by I.V.T.
4. (12 pts) Evaluate the limit (possibly as an infinite limit) or explain why it does not exist.

\[
\lim_{x \to -\infty} \cos(3^x + \pi)
\]

Since 3 > 1, \(3^x\) is small for \(x\) large and negative, i.e. \(\lim_{x \to -\infty} 3^x = 0\). By limit laws and the fact that \(\cos x\) is continuous, it follows that

\[
\lim_{x \to -\infty} \cos (3^x + \pi) = \cos (0 + \pi) = \cos (\pi) = -1.
\]

5. (12 pts) Evaluate the limit (possibly as an infinite limit) or explain why it does not exist.

\[
\lim_{x \to \pi/2} \frac{1}{1 - \cos x}
\]

Since \(\cos \frac{\pi}{2} = 1\) and \(\cos x\) is continuous, \(\cos x \to 1\) as \(x \to \pi/2\). Since \(\cos x \leq 1\) for all \(x\), \(1 - \cos x\) is small and positive for \(x\) near \(\pi/2\). Therefore,

\[
\lim_{x \to \pi/2} \frac{1}{1 - \cos x} = \infty.
\]

6. (12 pts) Find all horizontal and vertical asymptotes of the graph

\[
y = \frac{2x^2 - 5x + 9}{x^2 - 9}.
\]

The function \(f(x) = \frac{2x^2 - 5x + 9}{x^2 - 9}\) has infinite limits at \(x \to 3^+, x \to 3^-, x \to (-3)^+, x \to (-3)^-\), and \(\lim_{x \to \pm\infty} f(x) = \frac{2}{1} = 2\). Accordingly, the lines \(x = 3\) and \(x = -3\) are horizontal asymptotes, and \(y = 2\) is a horizontal asymptote.
7. (12 pts) Find the constant $C$ that makes the function

$$f(x) = \begin{cases} 
2x + C & x \leq 2 \\
\frac{x^2 - 3x + 2}{x - 2} & x > 2
\end{cases}$$

continuous on $(-\infty, \infty)$.

We have

$$\lim_{x \to 2^+} f(x) = \lim_{x \to 2^+} \frac{(x-1)(x-2)}{x-2} = \lim_{x \to 2^+} (x-1) = 1.$$ 

We need

$$\lim_{x \to 2^-} f(x) = 1,$$

so

$$4 + C = 1, \quad C = -3.$$ 

8. (12 pts) Fill in the missing numbers, functions or symbols indicated by the lettered boxes in the definition of

$$\lim_{x \to 3} \frac{1}{x+4} = \frac{1}{7}.$$ 

For every number $\varepsilon > 0$ there is a number $\delta > 0$ such that

if

$$0 < |x - A| < B$$

then

$$|C - 1/7| < D.$$ 

A 3

B $\delta$

C $\frac{1}{x+4}$

D $\varepsilon$