1. (8 pts each part) (a) Find the domain of the function $f(x) = \sqrt{\log_2 x}$.

Need x in domain of $\log_2 x$, so $x \geq 0$, and $\log_2 x$ in domain of $\sqrt{x}$, so $\log_2 x \geq 0$. This second condition means $x \geq 1$. So the domain of $\sqrt{\log_2 x}$ is

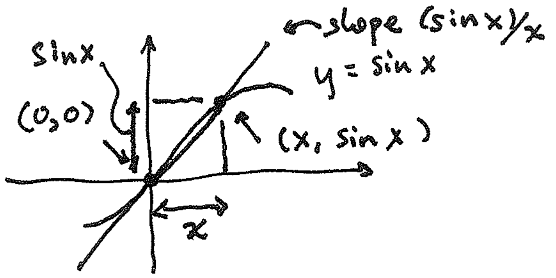
$[1, \infty)$.

(b) If $f(x)$ is one-to-one, find its inverse function. If $f(x)$ is not one-to-one, explain why.

It is one-to-one, and the inverse function is

$f^{-1}(x) = 2^{(x^2)}$.

2. (12 pts) Express the slope of the tangent line to the graph $y = \sin x$ at the point $(0, 0)$ as a limit. You do not have to evaluate the limit.



$$\lim_{x \to 0} \frac{\sin x}{x}$$

3. (12 pts) Which one of the intervals

$$(0, 1), \quad (1, 4) \quad \text{or} \quad (4, 9)$$

contains a root of the equation

$$\sqrt{x} = \frac{x - 1}{10 - x},$$

and why? You may take as known that only one of these intervals contains a root.

x	$\sqrt{x}$	$\frac{x-1}{10-x}$	$\sqrt{x} - \frac{x-1}{10-x}$
0	0	$-1/10$	$1/10$
1	1	0	1
4	2	$1/2$	$3/2$
9	3	8	-5

Since $\sqrt{x} - \frac{x-1}{10-x}$ is continuous, positive at $x=4$, and negative at $x=9$, it is zero somewhere in the interval $\boxed{(4,9)}$, by I.V.T..

4. (12 pts) Evaluate the limit (possibly as an infinite limit) or explain why it does not exist.

$$\lim_{x \to -\infty} \cos(3^x + \pi)$$

Since $3 > 1$, 3^x is small for x large and negative, i.e. $\lim_{x \to -\infty} 3^x = 0$. By Limit laws and the fact that $\cos x$ is continuous, it follows that

$$\lim_{x \to -\infty} \cos(3^x + \pi) = \cos(0 + \pi) = \cos(\pi) = -1 .$$

5. (12 pts) Evaluate the limit (possibly as an infinite limit) or explain why it does not exist.

$$\lim_{x \to \pi/2} \frac{1}{1 - \cos x}$$

Since $\cos \frac{\pi}{2} = 1$ and $\cos x$ is continuous, $\cos x \to 1$ as $x \to \pi/2$. Since $\cos x \leq 1$ for all x, $1 - \cos x$ is small and positive for x near $\pi/2$. Therefore,

$$\lim_{x \to \pi/2} \frac{1}{1 - \cos x} = \infty$$

6. (12 pts) Find all horizontal and vertical asymptotes of the graph

$$y = \frac{2x^2 - 5x + 9}{x^2 - 9}.$$

The function $f(x) = \dfrac{2x^2 - 5x + 9}{x^2 - 9}$ has infinite limits at $x \to 3^+$, $x \to 3^-$, $x \to (-3)^+$, $x \to (-3)^-$, and $\lim_{x \to \pm\infty} f(x) = 2$. Accordingly, the lines $x = 3$ and $x = -3$ are ~~horizontal~~ vertical asymptotes, and $y = 2$ is a horizontal asymptote.

7. (12 pts) Find the constant C that makes the function

$$f(x) = \begin{cases} 2x + C & x \leq 2 \\ \dfrac{x^2 - 3x + 2}{x - 2} & x > 2 \end{cases}$$

continuous on $(-\infty, \infty)$.

We have $\lim_{x \to 2^+} f(x) = \lim_{x \to 2^+} \frac{(x-1)(x-2)}{x-2} = \lim_{x \to 2^+} (x-1) = 1.$

We need $\lim_{x \to 2^-} f(x) = 1$, so $4 + C = 1$, $C = -3$.

8. (12 pts) Fill in the missing numbers, functions or symbols indicated by the lettered boxes in the definition of

$$\lim_{x \to 3} \frac{1}{x+4} = 1/7.$$

For every number $\epsilon > 0$ there is a number $\delta > 0$ such that

$$\text{if} \quad 0 < |x - \boxed{A}| < \boxed{B} \quad \text{then} \quad |\boxed{C} - 1/7| < \boxed{D}.$$

$\boxed{A}$ 3

$\boxed{B}$ δ

$\boxed{C}$ $\frac{1}{x+4}$

$\boxed{D}$ ϵ