1. (10 pts) Find the limit:

\[ \lim_{x \to 1} \frac{1 - x + \ln x}{(x - 1)^2} \]

By L'Hôpital's rule, (0/0),

\[ \lim_{x \to 1} \frac{-1 + \frac{1}{x}}{2(x-1)} = \lim_{x \to 1} \frac{-1}{2x} = \lim_{x \to 1} \frac{-1}{2} = \frac{-1}{2} \]

2. (10 pts) Using Newton’s method to approximate the solution of the equation \( \cos x = x \), with initial approximation \( x_0 = 1 \), what is the next approximation? Since you don’t have a calculator, write your answer as a formula, rather than evaluating it numerically.

We’re finding a root of \( f(x) = 0 \) where \( f(x) = \cos x - x \).

\[ x_1 = x_0 - \frac{f(x_0)}{f'(x_0)} = 1 - \frac{\cos(1) - 1}{-\sin(1) - 1} \]

3. (12 pts) Find all asymptotes, including slant asymptotes, to the graph

\[ y = \frac{(2x + 1)^3}{(x + 1)^2} \]

You do not have to sketch the graph.

\( m = \frac{8x^3 + 12x^2 + 6x + 1}{x^2 + 2x + 1} = 8x + \frac{-4x^2 - 2x + 1}{x^2 + 2x + 1} \)

\[ \lim_{x \to \pm \infty} y - 8x = \lim_{x \to \pm \infty} \frac{-4x^2 - 2x + 1}{x^2 + 2x + 1} = \lim_{x \to \pm \infty} \frac{-4 - \frac{2}{x} + \frac{1}{x^2}}{1 + \frac{2}{x} + \frac{1}{x^2}} = -4 \]

This shows \( y = 8x - 4 \) is a slant asymptote in both directions (\( x \to \infty \) and \( x \to -\infty \)). There is also a vertical asymptote at \( x = 1 \) where the denominator goes to 0.
4. (12 pts) Find the point or points on the parabola \( y = x^2 \) closest to the point \((0, 1)\) on the \( y \)-axis. Hint: you can simplify the problem by minimizing the square of the distance rather than the distance itself.

Square of distance from \((x, x^2)\) to \((0, 1)\) is
\[
d^2 = x^2 + (x^2 - 1)^2 = x^4 - x^2 + 1.
\]
We want to find its minimum. Critical numbers are at \(4x^3 - 2x = 0\), i.e.
\[
4x(x^2 - \frac{1}{2}) = 0; \quad x = 0 \text{ or } x = \pm \frac{1}{\sqrt{2}}.
\]
At \(x = 0\), \(d^2 = 1\).
At \(x = \pm \frac{1}{\sqrt{2}}\), \(d^2 = \frac{1}{4} - \frac{1}{2} + 1 = \frac{3}{4}\). So the closest points are \((\frac{1}{\sqrt{2}}, \frac{1}{2})\) and \((-\frac{1}{\sqrt{2}}, \frac{1}{2})\).

5. (12 pts) Find \(f(x)\) if \(f''(x) = x + \sin x\), \(f'(0) = 0\), \(f(0) = 2\).

\[
f(x) = \frac{x^2}{2} - \cos x + C_1 \quad f'(0) = 0 \implies C_1 = 1, \text{ so}
\]
\[
f'(x) = \frac{x^2}{2} - \cos x + 1.
\]
Then \(f(x) = \frac{x^3}{6} - \sin x + x + C_2\). \(f(0) = C_2 = 2, \text{ so}
\]
\[
f(x) = \frac{x^3}{6} - \sin x + x + 2.
\]

6. (12 pts) Evaluate the integral:

\[
\int_2^4 \frac{x^2 - 1}{x} \, dx
\]
\[
\int_2^4 x - \frac{1}{x} \, dx = \left[ \frac{x^2}{2} - \ln x \right]_2^4 = (8 - \ln 4) - (2 - \ln 2) = 6 - \ln 2, \text{ since } \ln 4 = 2 \ln 2.
\]
7. (10 pts) Evaluate the integral:

\[ \int_0^3 \sqrt{9-x^2} \, dx \]

The integral is the area of \( \frac{1}{4} \) of this circle:

\[ \int_0^3 \sqrt{9-x^2} \, dx = \frac{1}{4} \pi (3)^2 = \frac{9\pi}{4} . \]

8. (12 pts) Find the derivative \( f'(x) \), where

\[ f(x) = \int_0^{x^2} \tan(\sqrt{u}) \, du \]

Let \( g(x) = \int_0^x \tan(\sqrt{u}) \, du \). Then \( g'(x) = \tan(\sqrt{x}) \) by F.T.C.

Use chain rule to find

\[ f'(x) = \frac{d}{dx} g(x^2) = g'(x^2) \cdot 2x = 2x \tan \sqrt{x^2} = 2x \tan |x| . \]

9. (10 pts) Which is greater: the Riemann sum

\[ \frac{1}{3} + \frac{1}{1+1/3} + \frac{1}{1+2/3} \cdot \frac{1}{3} = 47/60 , \]

or the integral

\[ \int_1^2 \frac{1}{x} \, dx = \ln 2 , \]

and why?

This is a left-endpoint Riemann sum approximation to the area under a decreasing function, so it is greater than the exact area, given by the integral:

\[ \frac{47}{60} > \ln 2 . \]