Name _____ Section time & instructor _____

Student ID_____

Math 1A—Calculus, Fall 2010—Haiman
Final Exam

Instructions:

• Write your name, ID number and discussion section time and instructor's name at the top of this page. Do not look at the other pages until the signal to start is given.

• You may use one sheet (written on both sides) of prepared notes. No other notes, books, calculators, computers, cell phones, audio players, or other aids may be used.

• Use your own scratch paper for preliminary work, then write your solutions on the exam paper. Hand in only the exam paper itself.

• Write enough steps or words of explanation so that we can understand how you arrived at your answers. An answer that is just a number or a formula, without any explanation, will not receive partial credit if incorrect, and may not receive full credit even if correct.

• There are 22 questions, 100 total points.

Scores			
1		12	
2		13	
3		14	
4		15	
5		16	
6		17	
7		18	
8		19	
9		20	
10		21	
11		22	
Total:			

1. [4] Simplify $\cos(\sin^{-1}(2x))$ and state its domain.

2. [4] At which points is the function $f(x) = \begin{cases} 1/(1-e^x) & x < 0 \\ -x & 0 \le x < 1 \\ \cos \pi x & x \ge 1 \end{cases}$ (a) continuous, (b) continuous from the right, (c) continuous from the left, (d) neither?

3. [4] Differentiate $\ln(e^{\sqrt{2}\,x} + e^{-\sqrt{2}\,x})$.

4. [5] Find the point (a, b) on the graph $y = e^x$ where its tangent line passes through $(0, 0)$.

5. [4] Use a linear approximation or differentials to estimate $(8.15)^{2/3}$.

6. [4] Find the limit, either finite or infinite, or explain why it does not exist.

$$\lim_{x \to \pi/2} \frac{e^x - 1}{\cos x}$$

7. [4] Find the limit, either finite or infinite, or explain why it does not exist.

$$\lim_{x \to \infty} \frac{\ln(x+2)}{\ln(x+1)}$$

8. [4] Find the limit, either finite or infinite, or explain why it does not exist.

$$\lim_{x \to 1} (x+1)^{x-1}$$

9. [5] If $x^3 + y^3 = xy + 2$, find dy/dx in terms of x and y.

10. [5] If $-1 \leq f'(x) \leq 1$ for all x, and $f(1) = 5$, what can you conclude about the value of $f(4)$?

11. [5] Find all local maxima and minima of the function $f(x) = \dfrac{x}{x^2 + 9}$.

12. [5] Find the largest possible perimeter of a rectangle with lower-left corner at $(0, 0)$ and upper-right corner on the arc of the curve $xy = 4$ between $(1, 4)$ and $(4, 1)$.

13. [5] Alice is walking east and Bob is walking west along opposite sides of a street 10 m wide. If each walks at a speed of 2 m/s, how fast is the distance between them decreasing when Alice is 30 m west of Bob?

14. [4] For what values of A is the graph of $\cos x + Ax^2$ concave upward at every point?

15. [4] Show that $\int (\ln x)^2\,dx = x(\ln x)^2 - 2\,x\ln x + 2\,x + C$.

16. [5] Evaluate the integral $\displaystyle\int_0^2 |x(x-1)|\,dx$.

17. [5] Evaluate the indefinite integral $\displaystyle\int \frac{x^3}{x^2+1}\,dx$.

18. [5] Evaluate the integral $\displaystyle\int_0^{\pi/4} \tan x\,dx$.

19. [5] Find the area of the region enclosed by the curve $xy = 3$ and the line $x + y = 4$.

20. [5] Let R be the region bounded by the x-axis, the line $x = e$, and the graph of $y = \ln x$. Set S be the solid obtained by rotating R about the y-axis.

Set up, but do not evaluate, an integral which gives the volume of S using the method of slices.

21. [5] Set up, but do not evaluate, an integral which gives the volume of the solid S in the previous problem using the method of cylindrical shells.

22. [4] Find the average value of $\sqrt{1 - x^2}$ on the interval $[-1, 1]$.