1. (10 pts) Find the limit:

$$\lim_{x\to 1}\frac{1-x+\ln x}{(x-1)^2}$$

By L'Hospital's rule, (0/0),

$$= \lim_{x\to 1} \frac{-1+1/x}{2(x-1)} = \lim_{x\to 1}\frac{-(x-1)/x}{2(x-1)} = \lim_{x\to 1}\frac{-1}{2x} = \frac{-1}{2}$$

2. (10 pts) Using Newton's method to approximate the solution of the equation $\cos x = x$, with initial approximation $x_0 = 1$, what is the next approximation? Since you don't have a calculator, write your answer as a formula, rather than evaluating it numerically.

We're finding a root of $f(x)=0$ where $f(x) = \cos x - x$.

$$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)} = 1 - \frac{\cos(1)-1}{-\sin(1)-1}.$$

3. (12 pts) Find all asymptotes, including slant asymptotes, to the graph

$$y = \frac{(2x+1)^3}{(x+1)^2}.$$

You do not have to sketch the graph.

$$y = \frac{8x^3+12x^2+6x+1}{x^2+2x+1} = 8x + \frac{-4x^2-2x+1}{x^2+2x+1}$$

$$\lim_{x\to\pm\infty} y-8x = \lim_{x\to\pm\infty}\frac{-4x^2-2x+1}{x^2+2x+1} = \lim_{x\to\pm\infty}\frac{-4-2/x+1/x^2}{1+2/x+1/x^2} = -4.$$

This shows $y=8x-4$ is a slant asymptote in both directions ($x\to\infty$ and $x\to-\infty$). There is also a vertical asymptote at $x=-1$ where the denominator goes to 0.

4. (12 pts) Find the point or points on the parabola $y = x^2$ closest to the point $(0, 1)$ on the $y$-axis. Hint: you can simplify the problem by minimizing the square of the distance rather than the distance itself.

Square of distance from $(x, x^2)$ to $(0, 1)$ is $d^2 = x^2 + (x^2-1)^2 = x^4 - x^2 + 1$. We want to find its minimum. Critical numbers are at $4x^3 - 2x = 0$, i.e. $4x(x^2 - 1/2) = 0$ : $x = 0$ or $x = \pm 1/\sqrt{2}$. At $x = 0$, $d^2 = 1$. At $x = \pm 1/\sqrt{2}$, $d^2 = \frac{1}{4} - \frac{1}{2} + 1 = 3/4$. So the closest points are $(1/\sqrt{2}, 1/2)$ and $(-1/\sqrt{2}, 1/2)$.

5. (12 pts) Find $f(x)$ if $f''(x) = x + \sin x$, $f'(0) = 0$, $f(0) = 2$.

$f'(x) = \frac{x^2}{2} - \cos x + C_1$ $\quad f'(0) = 0 \Rightarrow C_1 = 1$, so

$f'(x) = x^2/2 - \cos x + 1$.

Then $f(x) = \frac{x^3}{6} - \sin x + x + C_2$. $\quad f(0) = C_2 = 2$, so

$f(x) = x^3/6 - \sin x + x + 2$

6. (12 pts) Evaluate the integral:

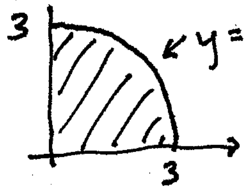
$$\int_2^4 \frac{x^2 - 1}{x}\, dx$$

$$\int_2^4 x - \frac{1}{x}\, dx = \frac{x^2}{2} - \ln x \Big]_2^4 = (8 - \ln 4) - (2 - \ln 2)$$

$$= 6 - \ln 2, \text{ since } \ln 4 = 2\ln 2.$$

7. (10 pts) Evaluate the integral:

$$\int_0^3 \sqrt{9-x^2}\,dx$$




$\leftarrow y=\sqrt{9-x^2}$ is $x^2+y^2=9$ : part of a circle of radius 3.

The integral is the area of $1/4$ of this circle:

$$\int_0^3 \sqrt{9-x^2}\,dx = \frac{1}{4}\pi(3)^2 = \frac{9\pi}{4}.$$

8. (12 pts) Find the derivative $f'(x)$, where

$$f(x)=\int_0^{x^2} \tan(\sqrt{u})\,du$$

Let $g(x)=\int_0^x \tan(\sqrt{u})\,du$. Then $g'(x)=\tan(\sqrt{x})$ by F.T.C.

Use chain rule to find

$$f'(x)=\frac{d}{dx}g(x^2) = g'(x^2)\cdot 2x = 2x\tan\sqrt{x^2} = 2x\tan|x|.$$

9. (10 pts) Which is greater: the Riemann sum

$$\frac{1}{1}\cdot\frac{1}{3}+\frac{1}{1+1/3}\cdot\frac{1}{3}+\frac{1}{1+2/3}\cdot\frac{1}{3}=47/60,$$

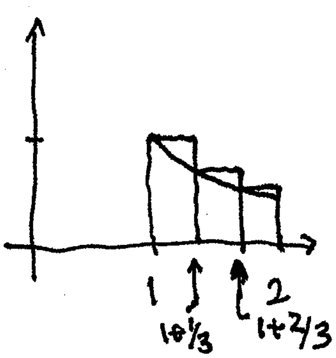
$$\frac{1}{x_1^*}\Delta x_1+\frac{1}{x_2^*}\Delta x_2+\frac{1}{x_3^*}\Delta x_3$$

with all $\Delta x_i = 1/3$, $x_1^*=1$, $x_2^*=1+1/3$, $x_3^*=1+2/3$

or the integral

$$\int_1^2 \frac{1}{x}\,dx = \ln 2,$$

and why?




This is a left-endpoint Riemann sum approximation to the area under a decreasing function, so it is greater than the exact area, given by the integral:

$$\frac{47}{60} > \ln 2$$