1. [10 pts] What is the geometric relationship between the graphs of $f(x) = \sqrt{x-3}+1$ and $g(x) = \sqrt{x} - 1$?

The graph of $g(x)$ is obtained by shifting the graph of $f(x)$ 3 units left and 2 units down.

2. [12 pts] Find the inverse function of $f(x) = \ln(2+\sqrt{x})$. What are the domain and range of $f(x)$ and of its inverse function?

To find $f^{-1}(x)$ set $x = \ln(2+\sqrt{y})$ and solve for y:

$2+\sqrt{y} = e^x$; $\sqrt{y} = e^x - 2$ $y = (e^x-2)^2$, i.e. $f^{-1}(x) = (e^x-2)^2$

The domain of f is $[0,\infty)$ and its range (since f is increasing and $\lim_{x\to\infty} f(x) = \infty$) is $[\ln 2, \infty)$.

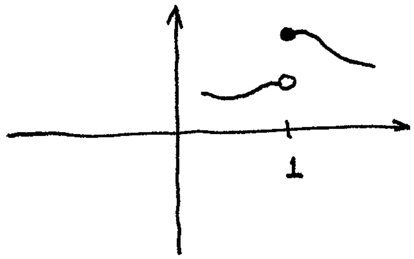
So $f^{-1}(x)$ has domain $[\ln 2, \infty)$ and range $[0,\infty)$.

(This is true even though the formula $(e^x-2)^2$ appears to be defined for all x, because $(e^x-2)^2$ is not 1-1 when its domain is taken to be $\mathbb{R}$.)

3. [10 pts] Is $5^{\log_2 3}$ equal to $3^{\log_2 5}$? Justify your answer.

Yes. Both are equal to $2^{(\log_2 3)(\log_2 5)}$.

4. [10 pts] Sketch a graph of a function $f(x)$ such that $\lim_{x\to 1^-} f(x)$ and $\lim_{x\to 1^+} f(x)$ both exist, and f is continuous from the from the right at $x = 1$, but not continuous at $x = 1$.



5. [12 pts] Find

$$\lim_{x\to 2} \frac{x-2}{x-4/x}$$

$$\lim_{x\to 2} \frac{x-2}{x-4/x} = \lim_{x\to 2} \frac{x(x-2)}{x^2-4} = \lim_{x\to 2} \frac{x}{x+2} = \frac{2}{2+2} = \frac{1}{2}.$$

6. [12 pts] Find all vertical and horizontal asymptotes to the graph

$$y = \frac{2x^2}{x-3x^2}.$$

$\lim_{x\to\infty} \frac{2x^2}{x-3x^2} = \lim_{x\to\infty} \frac{2}{1/x - 3} = \frac{2}{3}$, and $\lim_{x\to-\infty} \frac{2x^2}{x-3x^2} = 2/3$ also. So $y = \frac{2}{3}$ is the only horizontal asymptote.

Factoring the denominator, $y = \frac{2x^2}{x(1-3x)} = \frac{2x}{1-3x}$.

So $x = \frac{1}{3}$ is the only vertical asymptote.

At $x=0$, y is undefined, so the graph has a missing point there, but not a vertical asymptote, since $\lim_{x\to 0} \frac{2x^2}{x-3x^2}$ exists (it's $=0$).

7. [12 pts] Find the tangent line to the curve $y = 2x^3 - 3x$ at the point $(1, -1)$.

$y' = 6x^2 - 3$ at $x = 1$ gives slope 3.

So the tangent line is $y - (-1) = 3(x-1)$, or

$y = 3x - 4$.

8. [10 pts] Differentiate $3e^{2x} + 4e^{-x}$

$(e^{2x})' = (\ln e^2) e^{2x} = 2e^{2x}$, $(e^{-x})' = \ln(e^{-1}) e^{-x} = -e^{-x}$,

so $(3e^{2x} + 4e^{-x})' = 6e^{2x} - 4e^{-x}$

9. (a) [4 pts] Show that if $1 - \epsilon/5 < x < 1 + \epsilon/5$, then $2 - \epsilon < 5x - 3 < 2 + \epsilon$.

Multiplying $1 - \epsilon/5 < x < 1 + \epsilon/5$ by 5, then subtracting 3, gives

$$2 - \epsilon < 5x - 3 < 2 + \epsilon$$

(b) [8 pts] For what function $f(x)$ and numbers a and L does part (a) prove that $\lim_{x \to a} f(x) = L$?

Part (a) proves that

$$\lim_{x \to 1} (5x - 3) = 2$$

(the δ in the ϵ-δ definition is $\epsilon/5$ here).