1. Find the domain and range of the function

\[ f(x) = \frac{1}{(x - 2)^2}. \]

The domain is \( x \neq 2 \). The range is \( (0, \infty) \), since \( f(x) \) is always positive, approaches 0 for large \( x \), and approaches \( \infty \) for \( x \) approaching 2.

2. Express the function

\[ u(t) = \frac{\cos t}{1 + \cos t} \]

as a composite \( f \circ g \) of two other functions.

\[ u = f \circ g \] for \( f(t) = t/(1 + t) \), \( g(t) = \cos t \).

3. An exponential function \( f(x) = Ca^x \) has \( f(1) = 10 \) and \( f(3) = 40 \). Find the constants \( C \) and \( a \).

\[ \frac{f(3)}{f(1)} = a^3/a = a^2 = 4, \text{ so } a = 2. \] Plugging in \( x = 1 \) shows that \( C = 5. \)

4. Evaluate the limit, if it exists (possibly as an infinite limit).

\[ \lim_{x \to 3} \frac{\sqrt{x + 1} - 2}{x - 3}. \]

\[ \lim_{x \to 3} \frac{\sqrt{x + 1} - 2}{x - 3} = \lim_{x \to 3} \frac{x + 1 - 4}{(x - 3)(\sqrt{x + 1} + 2)} = \lim_{x \to 3} \frac{1}{\sqrt{x + 1} + 2} = 1/4. \]

5. Evaluate the limit, if it exists (possibly as an infinite limit).

\[ \lim_{x \to \pi/2} \tan x \]

The limit doesn’t exist, not even as an infinite limit, since \( \lim_{x \to (\pi/2)^-} \tan x = +\infty \) and \( \lim_{x \to (\pi/2)^+} \tan x = -\infty. \)

6. Evaluate the limit, if it exists (possibly as an infinite limit).

\[ \lim_{x \to \infty} \tan^{-1}(x^2 - x^4) \]

\( \lim_{x \to \infty} (x^2 - x^4) = -\infty, \) so \( \lim_{x \to \infty} \tan^{-1}(x^2 - x^4) = \lim_{x \to -\infty} \tan^{-1}(x) = -\pi/2. \)

7. In the definition of the limit

\[ \lim_{x \to 1} (5 - 3x) = 2, \]
find a value of $\delta$ that works for $\varepsilon = 0.1$.

Any $\delta$ less than or equal to 0.1/3 works. If $|x - 1| < 0.1/3$, then $|(5 - 3x) - 2| = |3 - 3x| = |3x - 3| = 3|x - 1| < 0.1$.

8. Prove that there is at least one real solution of the equation $xe^x = 1$.

Let $f(x) = xe^x$. It is a continuous function. Observe that $f(0) = 0$, and $f(1) = e$. Since $0 < 1 < e$, the intermediate value theorem shows that $f(c) = 1$ for some $c \in (0, 1)$.

9. For what value of the constant $c$ is the function

$$f(x) = \begin{cases} x + c & \text{if } x \leq -1 \\ x^2 - c & \text{if } x > -1 \end{cases}$$

continuous on $(-\infty, \infty)$?

The only possible discontinuity is at $x = -1$. Now, $f(-1) = \lim_{x \to (-1)^-} f(x) = -1 + c$, and $\lim_{x \to (-1)^+} f(x) = 1 - c$. To make $f(x)$ continuous at $x = -1$, we must have $-1 + c = 1 - c$, thus $c = 1$.

10. Differentiate the function

$$f(x) = (\sqrt{x} - 1)(e^x + x)$$

$$f'(x) = (\sqrt{x} - 1)'(e^x + x) + (\sqrt{x} - 1)(e^x + x)' = \frac{1}{2}x^{-1/2}(e^x + x) + (\sqrt{x} - 1)(e^x + 1).$$

11. Differentiate the function

$$f(x) = \frac{1}{x^3 - x + 2}$$

$$f'(x) = \frac{-(x^3 - x + 2)'}{(x^3 - x + 2)^2} = \frac{-3x^2 + 1}{(x^3 - x + 2)^2}.$$  

12. Find an equation of the tangent line to the curve

$$y = \sqrt{x}$$

at the point $(4, 2)$.

The slope is $y'(4) = (1/2)4^{-1/2} = 1/4$. The line has an equation of the form $y = (1/4)x + c$. To find $c$, use $2 = (1/4)4 + c$ at $(4, 2)$, so $c = 1$. Hence the desired equation is $y = x/4 + 1$.

13. Differentiate $e^x(\cos x + \sin x)$.

$$2e^x \cos x$$