

Name _____

Student ID Number _____

Discussion Section Instructor _____

(If you don't remember your section instructor's name, give section time instead.)

Instructions:

- Wait until you are told to begin before looking at the questions.
- After being told to start, put your name on each page, in case they get separated.
- Write answers in the space provided, and turn in only the exam paper. Show enough work so that we can see how you got your answers.
- You may use one prepared sheet of notes. No other notes, books or calculators are allowed.
- There are 15 questions, on both sides of the page. All questions have equal value.

| <i>For grading use only</i> | | | |
|-----------------------------|--|-------|--|
| 1 | | 9 | |
| 2 | | 10 | |
| 3 | | 11 | |
| 4 | | 12 | |
| 5 | | 13 | |
| 6 | | 14 | |
| 7 | | 15 | |
| 8 | | Total | |

1. Evaluate the limit if it exists (possibly as an infinite limit).

$$(a) \quad \lim_{x \rightarrow 1} \frac{1}{\ln x} \qquad (b) \quad \lim_{x \rightarrow 1} \frac{1}{(\ln x)^2}$$

2. Differentiate the function $y = \sin(\sin(\sin x))$.

3. Find (a) all local maxima and minima of the function

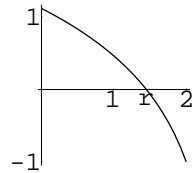
$$f(x) = \frac{x}{x^2 + 1},$$

and (b) the intervals of increase or decrease of $f(x)$.

4. Find the linear approximation to the function $f(x) = \ln x$ near $x = 2$.

5. If $y = e^{xy}$, express dy/dx in terms of x and y .

6. Suppose we use Newton's method to approximate the root r of the function whose graph is shown, using $x_1 = 1$ for the first approximation.



For the next approximation x_2 , decide whether $x_2 < r$ or $x_2 > r$. Justify your answer.

7. Find the largest area of a rectangle with horizontal and vertical sides, lower-left corner at the origin $(0, 0)$, and upper-right corner on the curve $y = e^{-x}$.

8. Find the limit.

$$\lim_{x \rightarrow \infty} x^{1/(1+\ln x)}$$

9. If $\int_a^x f(t) dt = x \ln x$ for all $x > 0$, find the function $f(x)$ and the constant a .

10. Evaluate the integral.

$$\int_0^2 x e^{-x^2} dx$$

11. Evaluate the indefinite integral.

$$\int \frac{(x+1)(x+2)}{x^2} dx$$

12. Sketch the region enclosed by the lines $x = 2$, $y = 2$ and the curve $xy = 1$, and find its area.

13. Find the average value of the function $f(x) = 1/x$ on the interval $[1, 3]$.

14. Find the volume of the circular cone obtained by rotating the triangle enclosed by the x and y axes and the line $x + y = 1$ about the y axis. (Solve the problem using integration. Do not just cite a formula you might already know for the volume of a cone.)

15. Set up, but do not evaluate, an integral for the volume of the solid obtained by rotating the region enclosed by the x axis, the line $x = 2$, and the curve $y = xe^{-x}$ about the y axis.