Prof. Haiman | Math 1A—Calculus | Fall, 2006

Quiz 9 solutions—version A

Name ______________________ Student ID Number ______________________

1. Find the limit, if it exists as a number or as an infinite limit.

$$\lim_{x\to 1} \frac{\sin \pi x}{\ln x}$$

This has the "0/0" form, so use L'Hospital's rule:

$$\lim_{x\to 1} \frac{\sin \pi x}{\ln x} = \lim_{x\to 1} \frac{\pi \cos \pi x}{1/x} = -\pi.$$

2. Find the limit, if it exists as a number or as an infinite limit.

$$\lim_{x\to 0^+} (\ln x + \ln(-\ln x)).$$

This has the "$\infty - \infty$" form. Rewrite it as

$$\lim_{x\to 0^+} (\ln x)(1 + \frac{\ln(-\ln x)}{\ln x}).$$

Using L'Hospital's rule in the "∞/∞" form, get

$$\lim_{x\to 0^+} \frac{\ln(-\ln x)}{\ln x} = \lim_{x\to 0^+} \frac{-1/(x\ln x)}{1/x} = 0.$$

Therefore,

$$\lim_{x\to 0^+} (\ln x)(1 + \frac{\ln(-\ln x)}{\ln x}) = \lim_{x\to 0^+} \ln x = -\infty.$$

3. Sketch a graph of the function, showing any horizontal, vertical or slant asymptotes:

$$y = \frac{x^2 + x + 1}{2x - 4}$$

Since the denominator vanishes at $x = 2$ while the numerator does not, the line $x = 2$ is a vertical asymptote. The form of the fraction suggests a slant asymptote with slope 1/2. To determine it precisely, compute

$$\lim_{x\to\infty} \frac{x^2 + x + 1}{2x - 4} - \frac{x}{2} = \lim_{x\to\infty} \frac{3x + 1}{2x - 4} = 3/2,$$

with the same limit as $x \to -\infty$. This shows that the line $y = (x+3)/2$ is a slant asymptote in both directions.