Prof. Haiman | Math 1A—Calculus | Fall, 2006

Quiz 8 solutions

Name ______________________ Student ID Number ______________________

For the function $f(x) = x^2 e^x$, find

(a) the critical points,

(b) the intervals of increase and decrease,

(c) the intervals of upward and downward concavity,

(d) the local minima and maxima,

(e) the inflection points.

Then make a reasonably accurate sketch of $y = f(x)$. Some of the following numbers might help: $\sqrt{2} \approx 1.4$, $e^0 = 1$, $e^{-.6} \approx .55$, $e^{-1} \approx .37$, $e^{-2} \approx .14$, $e^{-3} \approx .05$, $e^{-3.4} \approx .033$.

Since today's quiz has only one problem, it will be graded on a scale of 0 to 20, based on how many of the above features you correctly calculate, and how many you accurately display on your sketch.

Differentiate:

$$f'(x) = (x^2 + 2x)e^x = x(x+2)e^x.$$

(a) Critical points $f'(x) = 0$ are at $x = -2, 0$. (b) $f(x)$ is increasing for $x < -2$ and $x > 0$, decreasing on $(-2, 0)$. (d) $f(0) = 0$ is a local (and absolute) minimum, $f(-2) = 4e^{-2}$ is a local maximum.

Differentiate again:

$$f''(x) = (x^2 + 4x + 2)e^x.$$

We have $f''(x) = 0$ at $x = -2 \pm \sqrt{2}$, negative in between these two zeroes, positive elsewhere. Therefore (c) $f(x)$ is concave upward for $x < -2 - \sqrt{2}$ and $x > -2 + \sqrt{2}$, concave downward on $(-2 - \sqrt{2}, -2 + \sqrt{2})$, and (e) the inflection points are at $x = -2 \pm \sqrt{2}$.