

Quiz 7 solutions—version B

Name _____

Student ID Number _____

1. For the function $f(x) = x^3 - x + 1$, state what value of $f'(c)$ is guaranteed by the Mean Value Theorem to occur for some c in the interval $[0, 2]$, and find a c for which this value occurs.

Since $f(x)$ is differentiable on $[0, 2]$, the MVT guarantees the existence of a point c in this interval where $f'(c) = (f(2) - f(0))/2 = 3$. Compute $f'(x) = 3x^2 - 1$, and solve $3c^2 - 1 = 3$ to get $c = 2/\sqrt{3}$.

2. Find the maximum and minimum values of the function $f(x) = x^3 - 12x$ on the interval $[-3, 3]$.

The derivative is $f'(x) = 3x^2 - 12 = 3(x^2 - 4)$, giving critical points at $x = \pm 2$. Evaluating at the endpoints and the critical points,

$$f(-3) = 9$$

$$f(-2) = 16$$

$$f(2) = -16$$

$$f(3) = -9.$$

The minimum is $f(2) = -16$, and the maximum is $f(-2) = 16$.