

**Quiz 7 solutions—version A**

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1. Find the linear function of  $x$  which best approximates  $\sqrt[3]{x}$  for  $x$  close to 8, and use it to estimate  $\sqrt[3]{7}$ .

Taking  $f(x) = \sqrt[3]{x} = x^{1/3}$ , we have  $f'(x) = (1/3)x^{-2/3}$ ,  $f(8) = 2$ ,  $f'(8) = 1/12$ . The linear approximation is

$$y = 2 + \frac{x - 8}{12}.$$

This gives the estimate  $\sqrt[3]{7} \sim 23/12$  (by the way, to 4 decimal places,  $\sqrt[3]{7} = 1.9129\dots$ , compared to  $23/12 = 1.9166\dots$ ).

2. Find the maximum and minimum values of the function  $f(x) = xe^{-x}$  on the interval  $[0, 2]$ .

The derivative is  $f'(x) = (x - 1)e^{-x}$ , so  $x = 1$  is a critical point. Evaluating at the endpoints and the critical point,

$$f(0) = 0$$

$$f(1) = 1/e$$

$$f(2) = 2/e^2.$$

Since  $e > 2$ , we see that  $1/e > 2/e^2$ . Therefore the maximum is  $f(1) = 1/e$ , and the minimum is  $f(0) = 0$ .