Prof. Haiman | Math 1A—Calculus | Fall, 2004

Quiz 2 solution—version B

Name ______________________ Student ID Number ______________________

1. Calculate each limit, if it exists either as a number or as an infinite limit. If the limit doesn't exist, say so.

(a)

$$\lim_{x\to 1} \frac{x^3 - x^2 + x - 1}{x - 1} = \lim_{x\to 1} \frac{(x^2+1)(x-1)}{x-1} = 2.$$

(b)

$$\lim_{x\to 2^-} \frac{1}{x^2 - 4} = -\infty$$

2. (a) The fact that

$$\lim_{x\to 9} \sqrt{x} = 3$$

means that for every $\epsilon > 0$, there exists a $\delta > 0$ such that some condition holds. State that condition (as it applies to this specific limit).

$$0 < |x - 9| < \delta \quad \text{implies} \quad |\sqrt{x} - 9| < \epsilon,$$

or equivalently

$$9 - \delta < x < 9 + \delta,\ x \neq 9 \quad \text{implies} \quad 3 - \epsilon < \sqrt{x} < 3 + \epsilon.$$

(b) Find a δ that verifies the required condition if $\epsilon = 0.5$.

To get $2.5 < \sqrt{x} < 3.5$, need $(2.5)^2 < x < (3.5)^2$, so δ can be any positive number less than or equal to the smaller of $9 - (2.5)^2 = 2.75$ and $(3.5)^2 - 9 = 3.25$, that is, any $0 < \delta \leq 2.75$. For example, $\delta = 2$ would do.