Prof. Haiman Math 1A—Calculus Fall, 2004

Quiz 2 solution—version A

Name _____ Student ID Number _____

1. Calculate each limit, if it exists either as a number or as an infinite limit. If the limit doesn't exist, say so.

(a)

$$\lim_{x\to 2} \frac{x^3-4x}{x-2} = \lim_{x\to 2} \frac{x(x+2)(x-2)}{x-2} = 8.$$

(b)

$$\lim_{x\to 1} \frac{1}{x^2-1}$$

doesn't exist, since $1/(x^2-1) \to +\infty$ as $x \to 1^+$, but $1/(x^2-1) \to -\infty$ as $x \to 1^-$.

2. (a) The fact that

$$\lim_{x\to 1/2} \frac{1}{x} = 2$$

means that for every $\epsilon > 0$, there exists a $\delta > 0$ such that some condition holds. State that condition (as it applies to this specific limit).

$$0 < |x - 1/2| < \delta \quad \text{implies} \quad |(1/x) - 2| < \epsilon,$$

or equivalently

$$1/2 - \delta < x < 1/2 + \delta,\ x \neq 1/2 \quad \text{implies} \quad 2 - \epsilon < 1/x < 2 + \epsilon.$$

(b) Find a δ that verifies the required condition if $\epsilon = 0.1$.

To get $1.9 < 1/x < 2.1$, need $1/(2.1) < x < 1/(1.9)$, so δ can be any positive number less than or equal to the smaller of $1/2 - 1/(2.1) = 1/42$ and $1/(1.9) - 1/2 = 1/38$, that is, any $0 < \delta \leq 1/42$. For example, $\delta = .02$ would do.