Prof. Haiman

Math 1A—Calculus

Fall, 2006

Quiz 11 solutions—version B

Student ID Number _____

Name ____

1. Write down (you do not need to compute their values) two Riemann sums for the function $f(x) = \sin x$ on the interval $[0, \pi/2]$, using 5 equal subdivisions. Choose your Riemann sums so that one of them is an *upper* bound (that is, an overestimate) for the area A under the curve $y = \sin x$ on the interval $[0, \pi/2]$ and the other is a *lower* bound. Be sure to say which one is which.

Since sin x is increasing on $[0, \pi/2]$, the left-endpoint Riemann sum

$$\frac{\pi \sin 0}{10} + \frac{\pi \sin \pi/10}{10} + \frac{\pi \sin 2\pi/10}{10} + \frac{\pi \sin 3\pi/10}{10} + \frac{\pi \sin 4\pi/10}{10}$$

is a lower bound for A. The right-endpoint Riemann sum

$$\frac{\pi \sin \pi/10}{10} + \frac{\pi \sin 2\pi/10}{10} + \frac{\pi \sin 3\pi/10}{10} + \frac{\pi \sin 4\pi/10}{10} + \frac{\pi \sin \pi/2}{10}$$

is an upper bound.

2. Find the function F(x) such that $F''(x) = e^{2x}$, F(0) = 0, and F'(0) = 1.

Antidifferentiating twice, the general form of F(x) is $e^{2x}/4 + Cx + D$. Setting x = 0 in F(x) gives D = -1/4. Setting x = 0 in $F'(x) = e^{2x}/2 + C$ gives C = 1/2, so $F(x) = e^{2x}/4 + x/2 - 1/4$.