Quiz 11 solutions—version A

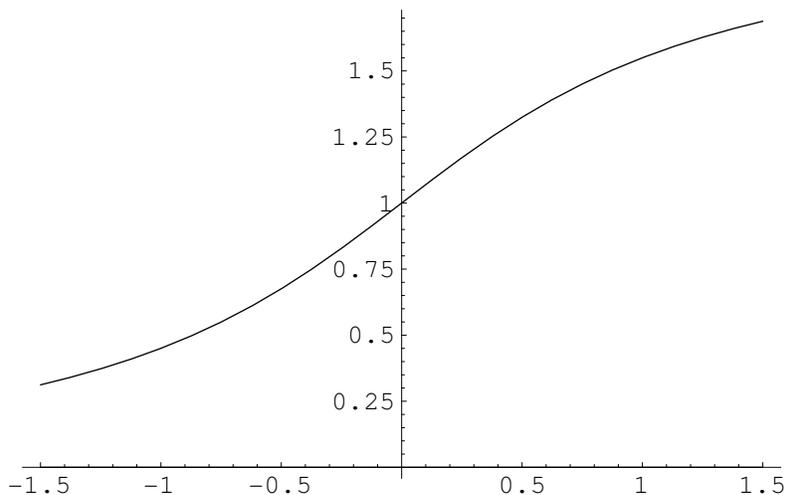Name ————————————   Student ID Number ————————————

1. On the graph $y = f(x)$ shown, indicate the region above the interval $[-1, 1]$ with area given *exactly* by the Riemann sum

$$S = \sum_{i=1}^{6} f(-1 + i/3)/3.$$

Is S greater or less than the area under the graph above the interval $[-1, 1]$?

The Riemann sum is the combined area of these rectangles, which is larger than the area under the graph since it is a right-endpoint Riemann sum and the function is increasing.
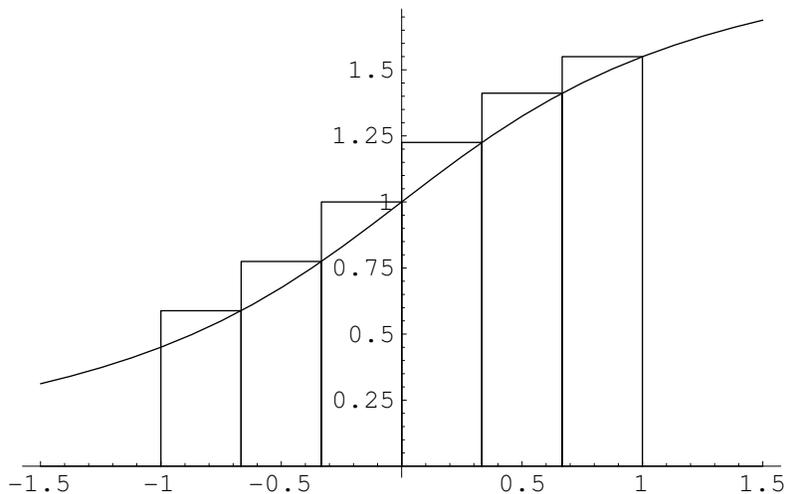
2. Find the function $F(x)$ such that $F''(x) = x$, $F(0) = -1$, and $F(1) = 0$.

Antidifferentiating twice, the general form of $F(x)$ is $x^3/6 + Cx + D$. Setting $x = 0$ gives $D = -1$. Setting $x = 1$ gives $C = 5/6$, so $F(x) = x^3/6 + 5x/6 - 1$.