Prof. Haiman | Math 1A—Calculus | Fall, 2006

Quiz 10 solutions—version B

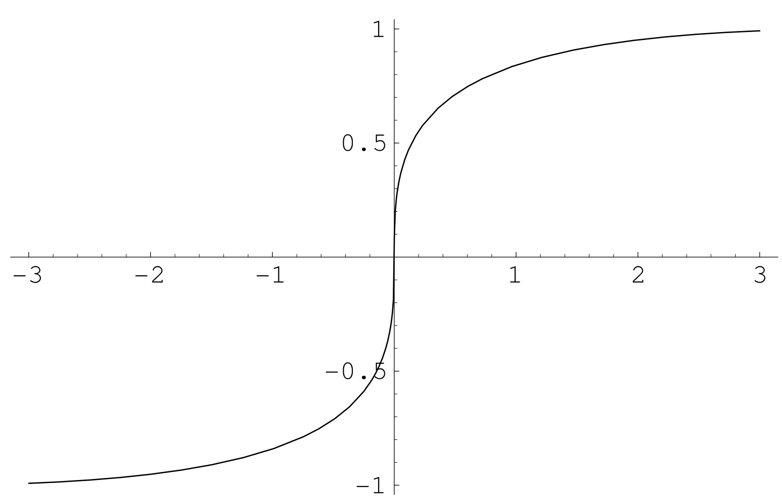
Name ____________________ Student ID Number ____________________

1. Use two steps of Newton's method, starting with $x_1 = 1$, to approximate $\sqrt[3]{2}$. Express answers as fractions p/q where p and q are integers.

Take $f(x) = x^3 - 2$, so $f'(x) = 3x^2$. First step: $f(x_1) = -1$, $f'(x_1) = 3$ gives $x_2 = 1-(-1)/3 = 4/3$. Second step: $f(x_2) = 10/27$, $f'(x_2) = 16/3$ gives $x_3 = 4/3-5/72 = 91/72$.

For comparison, $91/72 \approx 1.26389$, while $\sqrt[3]{2} \approx 1.25992$ to five decimal places. One more step of Newton's method would give the answer to this accuracy.

2. A computer plotted graph of the function $f(x) = \sin(\sqrt[3]{x})$ is shown below. In what way does the behavior of $f(x)$ for large x and for large negative x differ from what the graph suggests? Justify your answer.



The graph looks like it might be asymptotic to the lines $y = 1$ (as $x \to \infty$) and $y = -1$ (as $x \to -\infty$). However, since $\sqrt[3]{x}$ goes to $\pm\infty$ as $x \to \pm\infty$, the function $\sin(\sqrt[3]{x})$ actually oscillates infinitely many times through the range $[-1, 1]$ as $x \to \pm\infty$. This plot of $f(x)$ on a different scale gives a better idea of what happens.