1. Find $\frac{d^2}{dx^2}(\sec x)$.

$$\sec^3 x + \sec x \tan^2 x$$

2. Differentiate $x^{(e^x)}$.

$$x^{(e^x)}e^x(\ln x + \frac{1}{x})$$

3. If $h(x) = f(g(x))$ and $f(0) = 0, g(0) = 1, f'(0) = 2, g'(0) = 3, f'(1) = 4, g'(1) = 5$, find $h'(0)$.

$$h'(0) = f'(g(0))g'(0) = f'(1)g'(0) = 4 \cdot 3 = 12$$

4. If $x^2 + y^3 = 17$ and $dx/dt = 10$, find $dy/dt$ when $x = 3$.

Differentiating gives $2x \frac{dx}{dt} + 3y^2 \frac{dy}{dt} = 0$. Solve the original equation for $y$, getting $y = 2$ at $x = 3$. Then $60 + 12 \frac{dy}{dt} = 0$, so $\frac{dy}{dt} = -5$.

5. A cube is measured to be 6 cm on each side, with a possible error of± .5 cm. Use a linear approximation or differentials to estimate the error in computing the volume of the cube.

$$V = a^3, \quad dV = 3a^2 \, da.$$ With $a = 6$ and $da = .5$, get $dV = 54 \text{cm}^3$ (giving the answer as ±54 cm$^3$ is OK too).

6. Find all local and absolute minima and maxima of the function $f(x) = x^2(x + 6)$ on the interval $[-5, 3]$. Include local minima and maxima at the endpoints if there are any.

$$f'(x) = 3x^2 + 12x = 3x(x + 4)$$ has critical points at $x = -4, x = 0$, and sign $f'(x) > 0$ on $[-5, -4), f'(x) < 0$ on $(-4, 0), f'(x) > 0$ on $(0, 3]$. Then $f(-5) = 25$ is a local minimum, $f(-4) = 32$ is a local maximum, $f(0) = 0$ is the absolute minimum, and $f(3) = 81$ is the absolute maximum.

7. Verify that $f(x) = x^3 + x + 1$ satisfies the hypotheses of the Mean Value Theorem on the interval $[0, 2]$, and find all points $c$ for which the conclusion of the Mean Value Theorem holds.

$f(x)$ is continuous on $[0, 2]$ and differentiable on $(0, 2)$, with derivative $f'(x) = 3x^2 + 1$. The MVT guarantees that there is a point $c$ in $(0, 2)$ where $f'(c) = (9 - (-1))/2 = 5$. To find $c$, solve $3c^2 + 1 = 5$, $c = \pm 2/\sqrt{3}$. The negative solution is outside $(0, 2)$, so the only point is $c = 2/\sqrt{3}$.

8. Compute

$$\lim_{x \to 0} \frac{x + x^2}{e^x - e^{-x}}$$

Using L’Hospital’s rule,

$$\lim_{x \to 0} \frac{x + x^2}{e^x - e^{-x}} = \lim_{x \to 0} \frac{1 + 2x}{e^x + e^{-x}} = \frac{1}{2}.$$
9. Use the information below to sketch the graph of \( y = (x - 1)/x^2 \). Show any local or absolute maxima and minima and any inflection points by plotting them on your sketch and labelling them with their \( x \) and \( y \) coordinates.

- The domain of \( f(x) = (x - 1)/x^2 \) is \((-\infty, 0) \cup (0, \infty)\).
- \( \lim_{x \to 0^+} (x - 1)/x^2 = \lim_{x \to 0^-} (x - 1)/x^2 = -\infty \).
- \( \lim_{x \to \infty} (x - 1)/x^2 = \lim_{x \to -\infty} (x - 1)/x^2 = 0 \).
- \( y = 0 \) at \( x = 1 \), \( y < 0 \) on \((-\infty, 0) \cup (0, 1)\), and \( y > 0 \) on \((1, \infty)\).
- \( y' = (2 - x)/x^3 \); \( y' = 0 \) at \( x = 2 \), \( y' < 0 \) on \((-\infty, 0) \cup (2, \infty)\), and \( y' > 0 \) on \((0, 2)\).
- \( y'' = (2x - 6)/x^4 \); \( y'' = 0 \) at \( x = 3 \), \( y'' < 0 \) on \((-\infty, 0) \cup (0, 3)\), and \( y'' > 0 \) on \((3, \infty)\).

See answer to Chapter 4.5, Exercise 15 in the textbook.