Math 1A | Calculus | Fall, 2004

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First Midterm Exam—Solutions

1. Find the domain of the function $f(x) = \sqrt{x} - \sqrt{3-x}$.

Both x and $3-x$ must be positive, so the domain is $[0,3]$.

2. Find a formula for the inverse function $g(x)$ of the function

$$f(x) = e^{x^2+1}.$$

Sorry, this question was badly formulated. I should have specified that the domain of $f(x)$ is $[0,\infty)$. Since I didn't, you could correctly answer that the function is not one-to-one and therefore has no inverse.

If you assume that the domain of $f(x)$ is meant to be $[0,\infty)$, then solving $y = e^{x^2+1}$ for x gives $x = \sqrt{\ln(y)-1}$, so $g(x) = \sqrt{\ln(x)-1}$. (The domain of $g(x)$ is $[e,\infty)$, but it was not required that you specify it.)

3. Evaluate the limit, if it exists (possibly as an infinite limit).

$$\lim_{x\to 3^-} \frac{x-5}{x-3}.$$

The limit is $+\infty$, since both numerator and denominator are negative as $x \to 3^-$.

4. Evaluate the limit, if it exists (possibly as an infinite limit).

$$\lim_{x\to 1} \frac{x^3-1}{x^2-1}.$$

$$\lim_{x\to 1} \frac{x^3-1}{x^2-1} = \lim_{x\to 1} \frac{x^2+x+1}{x+1} = \frac{3}{2}.$$

5. Let $f(x) = 3 + 1/x$. In the definition of the limit $\lim_{x\to\infty} f(x) = 3$, if $\varepsilon = 1/5$, how large must N be to guarantee that $|f(x) - 3| < \varepsilon$ for all $x > N$?

We need $N \geq 5$. Then $x > 5$ gives $|f(x) - 3| = |1/x| = 1/|x| < 1/5$.

6. Show that the equation $x^4 - x - 1 = 0$ has at least one real solution in the interval $(1,2)$.

Let $f(x) = x^4 - x - 1$. Then $f(x)$ is continuous, $f(1) = -1$, and $f(2) = 13$. Since $-1 < 0 < 13$, there is at least one $c \in (1,2)$ such that $f(c) = 0$, by the intermediate value theorem.

7. Differentiate the function

$$f(x) = \frac{2x+1}{x+3}.$$

$$f'(x) = \frac{(x+3)\cdot 2 - (2x+1)\cdot 1}{(x+3)^2} = \frac{5}{(x+3)^2}.$$

8. Differentiate the function

$$f(x) = \sqrt{x}e^x.$$

$f'(x) = \frac{1}{2}x^{-1/2}e^x + x^{1/2}e^x = (\frac{1}{2}x^{-1/2} + x^{1/2})e^x.$

9. Find the values of x where the graph of $y = x^3 - 6x^2$ has a horizontal tangent line.

We must find the values of x where $y' = 0$. Now $y' = 3x^2 - 12x = 3x(x-4)$, so $x = 0$ or $x = 4$.