Quiz 8 Solution (Version A)

1. Find the limit

\[
\lim_{x \to 1} (\ln x) (\tan \frac{\pi x}{2})
\]

Now we have a 0/0 type limit and can apply L’Hospital’s rule to get

\[
\lim_{x \to 1} \frac{1/x}{-(\pi/2) \csc^2 \pi x/2} = -2/\pi.
\]

2. A rectangular box has height \( h \), width \( w \) and depth \( d \). Find the largest possible volume for the box if it is required that \( w = 2h \), and the total perimeter \( h + w + d \) is 3 m.

The constraints imply \( 3h + d = 3 \), so \( d = 3 - 3h \). The volume is

\[
V = hwd = h(2h)(3 - 3h) = 6h^2 - 6h^3.
\]

We are to maximize this on the interval \( 0 \leq h \leq 1 \).

\[
dV/dt = 12h - 18h^2 = 6h(2 - 3h)
\]

giving a critical point at \( h = 2/3 \), in addition to the endpoints \( h = 0, 1 \) of the domain. We have \( V = 0 \) at the endpoints, so the absolute maximum is \( V = (2/3)(4/3)(1) = 8/9 \) m\(^3\), with \( h = 2/3, w = 4/3, d = 1 \).