Math 172—Combinatorics—Spring 2010
Problem Set 8

Suggested study exercises: Chapter 7, Ex. 3, 5, 7, 8, 9, 10, 11, 13

Problems from the book: Chapter 7, Ex. 16 [express answer in terms of derangement numbers $D(n)$], 18, 20.

Additional problems:

A. Let $e(2n,k)$ denote the number of partitions of $[2n]$ with k cycles, all of even length. In class, we found the generating function

$$\sum_k e(2n,k)x^k = (2n-1)(2n-3)\cdots 3\cdot 1\cdot (x+2n-2)(x+2n-4)\cdots(x+2)x,$$

which generalizes Theorem 6.24 in your book. Use the above generating function and Lemma 6.13 in your book to find an explicit formula for $e(2n,k)$ in terms of $c(n,k)$.

B. In class, we found the ordinary generating function for Stirling numbers of the second kind, with k fixed,

$$\sum_{n=0} S(n,k)x^n = \frac{x^k}{(1-x)(1-2x)\cdots(1-kx)}.$$

(i) Use the generating function to derive the recurrence

$$S(n,k) = S(n-1,k-1) + k\,S(n-1,k).$$

(ii) Use a partial fraction expansion on the right-hand side to get an explicit formula for $S(n,3)$ for all n.

C. Find the number of weak compositions of 100 with four parts, all of which are less than or equal to 30. Express your answer in terms of binomial coefficients.