Math 172—Combinatorics—Spring 2010
Problem Set 6

Suggested study exercises: Chapter 8, Ex. 5, 6, 10, 11, 15, 16, 17, 18 (we'll discuss in class how to solve 15–18 without resorting to recurrences).

Problems from the book:

Chapter 8, Ex. 26 (hint: first show that every permutation p has a unique decomposition in one-line notation $p = j_1, \ldots, j_k$, where j_1 is an indecomposable permutation of $\{1, \ldots, m_1\}$, j_2 is an indecomposable permutation of $\{m_1 + 1, \ldots, m_2\}$, etc., for some k and some $m_1 < m_2 < \cdots < m_k = n$.)

Chapter 8, Ex. 32 (hint: set $t = -1$ in the generating function $\sum_{n,k} p(n, k)x^n t^k$).

Addtional Problems:

A. Let $q(n)$ denote number of weak compositions $(r_1, r_2, \ldots, r_n)$ of n with the property that r_i is a multiple of i for all i.

(a) Find the generating function $\sum_{n=0}^{\infty} q(n)x^n$.

(b) Deduce from (a) the relationship between $q(n)$ and $p(n)$, the number of partitions of the integer n.

(c) Find a direct combinatorial explanation, not using generating functions, of the relationship discovered in (b).

B. Do the exercise at the end of Section 2 of the Notes on Ordinary Generating Functions.

C. Let $c(n)$ be the number of compositions of n with odd parts.

(a) Find the generating function $\sum_{n=0}^{\infty} c(n)x^n$.

(b) By comparing the result with the Fibonacci generating function $F(x)$ in the solution to Chapter 8, Exercise 4, find a formula for $c(n)$ in terms of Fibonacci numbers.

D. Suppose that in your wallet you have 3 twenty-dollar bills, 5 tens, and 6 fives. Find a formula for the polynomial $P(x)$ whose coefficient of x^n is the number of ways you can make change for n dollars, under each of the following two assumptions:

(a) Two change combinations are considered the same if they contain the same number of bills of each denomination, i.e., we treat the bills of a given denomination as indistinguishable.

(b) We count change combinations as different if the individual bills used are different, i.e., we treat all individual bills as distinguishable (for instance, by looking at their serial numbers).