Math 172—Combinatorics—Spring 2010
Problem Set 5

Suggested study exercises: Chapter 6, Ex. 10 (I actually meant 10 instead of 9 on PS 4), 18, 19, 21, 25. Chapter 8, Ex. 1, 2, 3, 4, 7, 8, 12, 13.

Problems from the book:

Chapter 6, Ex. 38, 39, 41
Chapter 8, Ex. 23, 44

Addtional Problems:

A. (a) Show that the number of permutations of $[2n]$ in which every cycle has length 2 is equal to $(2n-1)(2n-3)\cdots 3\cdot 1$.

(b) Show that the number of permutations of $[2n+1]$ with one fixed point, and all other cycles having length 2, is equal to $(2n+1)(2n-1)(2n-3)\cdots 3\cdot 1$

B. Show that the number of permutations of $[2n]$ whose one-line notation $p_1\cdots p_{2n}$ satisfies $p_i > p_{i+1}$ for all odd $i \le 2n-1$, and $p_i < p_{i+2}$ for all odd $i \le 2n-3$ is equal to the number in problem A, part (a).

C. Let $f(n)$ be the number of partitions of n whose parts belong to $\{1,2\}$. Find the generating function $F(x) = \sum_{n=0}^{\infty} f(n)x^n$.