

**Math 172—Combinatorics—Spring 2010**  
**Problem Set 4**

Suggested study exercises: Chapter 6, Ex. 1, 2, 4, 5, 6, 7, 9 (in my copy of the book the solution to Ex. 10 is in the middle of the solution of Ex. 9), 13, 14, 17, 25.

Problems from the book:

Chapter 6: Ex. 33, 40 (find a bijection).

Additional Problems:

For the following problems, we define a *circular distribution* from  $X$  to  $Y$  to be a function  $f : X \rightarrow Y$ , together with an arrangement of the set  $f^{-1}(y)$  into a cycle for each  $y \in Y$ . (Informally, these are seating arrangements for a set  $X$  of people at a set  $Y$  of circular tables.)

A. Let  $|X| = n$ ,  $|Y| = k$ . Express the number equivalence classes of *surjective* circular distributions from  $X$  to  $Y$  in terms of counting numbers  $c(n, k)$ ,  $S(n, k)$ ,  $p(n, k)$ , binomial coefficients, etc., in each of the cases: (i)  $X$  and  $Y$  have distinguishable elements; (ii)  $X$  has indistinguishable elements; (iii)  $Y$  has indistinguishable elements; (iv) both  $X$  and  $Y$  have indistinguishable elements.

B. Same as problem A, but without the restriction to surjective distributions (some answers may have to be expressed as summations).

C. Find the number of permutations of  $[5]$  with each possible cycle type, and check that your answers agree with the values of  $c(5, k)$  for  $k = 1, \dots, 5$ .