Math 172—Combinatorics—Spring 2010
Problem Set 3

Suggested study exercises: Chapter 5, Ex. 6, 7, 8, 9, 11, 12, 13, 14.

Problems from the book:

Chapter 5: Ex. 26, 27 (it's probably easiest first to solve 27 and use the result to solve 26, although strictly speaking this doesn't give a bijective proof in 26).

Addtional Problems:

A. Show that the number of partitions λ of $m^2 + n$ such that the Durfee square (see Ex. 8) of λ has size $m \times m$ is equal to

$$\sum_{k=0}^{n} p_{\leq m}(k) p_{\leq m}(n-k),$$

where $p_{\leq m}(k) = p(k,0) + p(k,1) + \cdots + p(k,m)$ denotes the number of partitions of k with at most m parts.

Then use this result to solve Chapter 5, Exercise 25.

B. Show that the number of partitions of n with k distinct parts is equal to the number of partitions of n with largest part k and in which every number from 1 through k occurs as a part at least once.

C. Show that the numbers in Problem B are also equal to $p_{\leq k}(n - \binom{k+1}{2})$ (same notation as in Problem A).

D. Find the conjugates of the partitions $(4,3,3,2)$, $(5,2,1,1)$ and $(9,5,4,3) = (4+5, 3+2, 3+1, 2+1)$. Then find the general rule of which this is a special case.