Math 172—Combinatorics—Spring 2010
Problem Set 1

Homework ground rules: if a problem has a numerical or one-word answer, give some indication of your reasoning in addition to the answer.

You may discuss the problems with other students, but you must write your solutions independently, without consulting notes from discussions with others. You also may not consult solutions to similar problems from external sources (e.g., previous year's classes, other textbooks, solutions found on the internet, etc.).

Suggested study exercises (with solutions in the book, not to hand in; some of these may be harder than homework problems I would assign): Chapter 1, Ex. 3, 6, 9 [note: in the solution, "7×7 square" should be "7×1 rectangle" instead], 11; Chapter 2, Ex. 3, 4, 7; Chapter 3, Ex. 1-15, 18, 19, 23.

Problems from the book:

Chapter 2: Ex. 17

Chapter 3: Ex. 25, 26, 27, 38, 41

Additional problems:

A. Suppose given a subset $X \subseteq \{1, \ldots, 30\}$ of size $|X| = 21$. Prove that X must contain three elements $x, x + 10, x + 20$ for some x.

B. Find the number of ways to distribute 9 different candies to three children if the oldest child gets 2 candies, the middle child gets 3 candies, and the youngest child gets 4 candies.

C. (a) In a club with 20 members, how many ways are there to choose two committees, with 3 and 4 members respectively, and no one on both committees?

(b) In a club with 20 members, how many ways are there to choose a committee of 7 members and a subcommittee of 3 members from the larger committee.

(c) Generalizing the first two parts of this problem, find a combinatorial proof (i.e., not using the formulas for binomial and multinomial coefficients) of the identities

$$\binom{n}{m}\binom{n-m}{k} = \binom{n}{k}\binom{n-k}{m} = \binom{n}{m, k, n-m-k}$$

D. Let H be a set of ten integers between 1 and 99. Prove that H must have two disjoint, nonempty subsets A and B such that the sum of the numbers in A is equal to the sum of the numbers in B. [Hint: first consider the problem without the requirement that A and B be disjoint.]