Math 113, Summer 2015 Prof. Haiman

Problem Set 7

This assignment is due on *Monday*, Aug. 10.

It covers class material for Aug 3–6 and reading from Sections 3.3, 7.3–4, 9.1–2, and parts of 7.5 as indicated in the 'Guide to Field Theory' notes.

Section 3.3: 2. Additional problem: show that if A is an $m \times n$ matrix with entries in K, then (a) the columns of A are linearly independent vectors in K^m if and only if the equation $A\mathbf{x} = 0$ has only the trivial solution $\mathbf{x} = 0$ in K^n; (b) the columns of A span K^m if and only if the equation $A\mathbf{x} = \mathbf{b}$ has a solution, for every $\mathbf{b} \in K^m$; (c) the columns of A form a basis of K^m if and only if $A\mathbf{x} = \mathbf{b}$ has a unique solution for every $\mathbf{b} \in K^m$.

Section 7.3: 4, 7, 11, 13. In Problem 4, you need to show (i) that the set of ratios $p(a_1, \ldots, a_n)/q(a_1, \ldots, a_n)$, where $q(a_1, \ldots, a_n) \neq 0$, is a subfield M of L, and (ii) that every subfield of L which contains K and the elements a_i must contatin M.

Section 7.4: 1, 2, 4, 6. In Problem 6, you need to show that $\mathrm{Fix}(H)$ contains K, and that it is a subfield of L (i.e., a unital subring which is a field).

Section 7.5: Do the part of Problem 5 in its first sentence.

Section 9.1: 4. Hint: calculate powers of $\sqrt{2} + \sqrt{3}$ in $\mathbb{Q}(\sqrt{2}, \sqrt{3})$ to find the minimal polynomial $p(x) \in \mathbb{Q}[x]$ of $\sqrt{2} + \sqrt{3}$.

Section 9.2: 2. Hint: you can show that $f(x) = x^6 - 3$ is irreducible by using its factorization over $\mathbb{Q}(3^{1/6})$, since any factor of $f(x)$ over $\mathbb{Q}$ must be a product of some of the factors over $\mathbb{Q}(3^{1/6})$. You can also do it using Eisenstein's criterion.