

Math 110 Assignment 9

(I) *Exercises.*

Axler Chapter 7: 1, 2, 3, 4, 5, 16, 21

(II) *Problems.* Due Friday, April 13 by 3pm at the location your GSI has specified for turning in homework.

Recall from Lecture 21 (if you missed this lecture, you may want to get notes from a classmate) that an $n \times n$ matrix Q over \mathbb{R} is called *orthogonal* if its columns form an orthonormal basis of \mathbb{R}^n with the Euclidean inner product; a matrix over \mathbb{C} satisfying the analogous condition is called *unitary*.

(10/10) Prove that the following properties of a square matrix over \mathbb{R} or \mathbb{C} are equivalent:

- (a) Q is unitary (if $\mathbb{F} = \mathbb{C}$) or orthogonal (if $\mathbb{F} = \mathbb{R}$).
- (b) QQ^* is the identity matrix.
- (c) The conjugate transpose Q^* is unitary (if $\mathbb{F} = \mathbb{C}$) or orthogonal (if $\mathbb{F} = \mathbb{R}$).
- (d) The rows of Q form an orthonormal basis of \mathbb{F}^n .