Prof. Haiman Spring 2012

Math 110 Assignment 7

(I) *Exercises.*

Axler Chapter 6: 2, 3, 4, 5, 6, 9, 10, 13, 14

(II) *Problems.* Due Friday, Mar. 16 by 3pm at the location your GSI has specified for turning in homework.

Initially I assigned Axler 6.8 (for $\mathbb{F} = \mathbb{R}$), but this problem seems to be too hard, so I'm changing it.

1. (3/10) Prove the identity

$$b\,\|u+av\|^2 + a\,\|u-bv\|^2 = (a+b)(\|u\|^2 + a\,b\,\|v\|^2)$$

for all vectors u, v in an inner product space V, and all real scalars $a, b \in \mathbb{R}$. (The parallelogram identity, Axler Proposition 6.14, is the special case of this with $a = b = 1$.)

2. (3/10) Prove the identity

$$\|u+v+w\|^2 - \|u+v\|^2 - \|u+w\|^2 - \|v+w\|^2 + \|u\|^2 + \|v\|^2 + \|w\|^2 = 0$$

for all vectors u, v, w in an inner product space V. (The parallelogram identity is also a special case of this identity: take $w = -v$.)

3. (4/10) Let V be a vector space over $\mathbb{R}$, and suppose a norm $\|\cdot\|\colon V \to \mathbb{R}$ is given which satisfies the conditions in Axler, Exercise 6.8, and also the generalized parallelogram identities in Problems 1 and 2, above. Prove that there is an inner product on V such that $\|v\| = \sqrt{\langle v, v\rangle}$ for all $v \in V$.

Hint: find an expression for $\langle u, v\rangle$ in terms of $\|u+v\|$, $\|u\|$ and $\|v\|$ which holds in any inner product space over $\mathbb{R}$, and use this to define the inner product in terms of the given norm. In this version of the problem, you will not need to use the assumption that $\|\cdot\|$ satisfies the triangle inequality.