Prof. Haiman Spring 2012

Math 110 Assignment 5

(I) *Exercises.*

Axler Chapter 3: 14, 15; Chapter 10: 1, 2, 4; Chapter 4: 1, 5; Chapter 5: 1, 2, 3, 5

(a) Show that the map $S\colon P_3(\mathbb{R}) \to \mathbb{R}^4$ defined by $Sp(z) = (p(0), p(1), p'(0), p'(1))$ is linear and injective. Hint: show that $p(a) = p'(a) = 0$ if and only if $(z-a)^2$ is a factor of $p(z)$.

(b) Using Exercise (a), show that for all real numbers a, b, c, d there is a unique polynomial $p(z) \in P_3(\mathbb{R})$ such that $p(0) = a$, $p(1) = b$, $p'(0) = c$, and $p'(1) = d$. In other words, there is a unique arc of a cubic curve $y = a_3x^3 + a_2x^2 + a_1x + a_0$ with prescribed endpoints $(0, a)$ and $(1, b)$, and prescribed slopes c, d at the endpoints. Such arcs are called *cubic splines.*

(II) *Problems.* Due Friday, Mar. 2 by 3pm at the location your GSI has specified for turning in homework.

1. Axler Chapter 5 Exercise 12

2. We proved in class that if $r_1, \ldots, r_{d+1}$ are distinct elements of $\mathbb{F}$, then the evaluation map

$$T\colon P_d(\mathbb{F}) \to \mathbb{F}^{d+1}$$

defined by $Tp(z) = (p(r_1), \ldots, p(r_{d+1}))$ is invertible (we did this for $\mathbb{F} = \mathbb{R}$ in the lecture, but the same proof is valid for any field $\mathbb{F}$). Using this result, prove that if $r_1, \ldots, r_{d+1}$ are distinct, then the matrix

$$A = \begin{pmatrix} 1 & 1 & \ldots & 1 \\ r_1 & r_2 & \ldots & r_{d+1} \\ r_1^2 & r_2^2 & \ldots & r_{d+1}^2 \\ \vdots & \vdots & & \vdots \\ r_1^d & r_2^d & \ldots & r_{d+1}^d \end{pmatrix} \in M_{(d+1)\times(d+1)}(\mathbb{F})$$

is invertible.