Prof. Haiman Spring 2012

Math 110 Assignment 4

(I) *Exercises.*

Axler Chapter 3: 17 (by relating it to properties of linear maps, rather than by working directly with the definitions of matrix operations), 19, 21, 22, 23, 25, 26.

(a) Show that the operator $T\,p(z) = p''(z) - 2zp'(z) + zp(z)$ is linear and maps $P_d(\mathbb{R})$ into $P_{d+1}(\mathbb{R})$.

(b) Find the matrix of the operator T in Exercise (a) from $P_4(\mathbb{R})$ to $P_5(\mathbb{R})$, relative to the standard monomial bases of these spaces.

(c) There is a linear map $S : P(\mathbb{F}) \to \mathbb{F}^\infty$ which sends a polynomial $a_0 + a_1 z + \cdots + a_d z^d$ to the sequence of its coefficients $(a_0, a_1, \ldots, a_d, 0, 0, \ldots)$. Is S invertible? Is it injective? Is it surjective?

(II) *Problems.* Due Friday, Feb. 24 by 3pm at the location your GSI has specified for turning in homework.

Let U and V be finite-dimensional vector spaces over $\mathbb{F}$, with bases $(u_1, \ldots, u_n)$ of U and $(v_1, \ldots, v_m)$ of V. For each $j = 1, \ldots, n$, Let $L_j \colon U \to \mathbb{F}$ be the map $L_j(a_1u_1 + \cdots + a_nu_n) = a_j$. For each $i = 1, \ldots, m$, let $K_i \colon \mathbb{F} \to V$ be the map $K_i(a) = av_i$.

(a) Verify that L_j and K_i are linear.

(b) What is the matrix $\mathcal{M}(K_iL_j)$, with respect to the given bases of U and V?

(c) Prove that the maps K_iL_j for all $i = 1, \ldots, m$ and $j = 1, \ldots, n$ form a basis of $\mathcal{L}(U, V)$.