Math 110—Linear Algebra
Fall 2009, Haiman
Problem Set 12

Due Monday, Nov. 23, at the beginning of lecture.

1. Let J_n denote the $n \times n$ matrix over $\mathbb{R}$ whose entries are all equal to 1.

(a) Show that $(1, 1, \ldots, 1)^t$ is an eigenvector of J_n. What is its eigenvalue?

(b) Find the dimension of the nullspace of J_n.

(c) Use (a) and (b) to show that J_n is diagonalizable, and find the diagonal matrix similar to J_n.

(d) Find the characteristic polynomial of J_n.

(e) Let $Z_n = J_n - I_n$ be the $n \times n$ matrix with zeroes on the diagonal and ones in all off-diagonal entries. Find $\det(Z_n)$, and show that Z_n is invertible for $n > 1$.

(f) Find the characteristic polynomial of Z_n.

(g) Find a quadratic polynomial $f(t)$ (with coefficients depending on n) such that $f(Z_n) = 0$.

(h) Use (g) to calculate the inverse of Z_n, expressed as a linear combination of Z_n and I_n. (This generalizes Problem Set 7, Problem 3.)

2. Let $T\colon V \to V$ be a linear operator, where V is finite dimensional. Suppose that $W_1, \ldots, W_k$ are T-invariant subspaces of V such that T_{W_i} is diagonalizable for each i. Prove that if $W_1 + \cdots + W_k = V$, then T is diagonalizable.

3. Section 5.4, Exercises 13 and 20.