Due Monday, Nov. 23, at the beginning of lecture.

1. Let $J_n$ denote the $n \times n$ matrix over $\mathbb{R}$ whose entries are all equal to 1.
   (a) Show that $(1, 1, \ldots, 1)^t$ is an eigenvector of $J_n$. What is its eigenvalue?
   (b) Find the dimension of the nullspace of $J_n$.
   (c) Use (a) and (b) to show that $J_n$ is diagonalizable, and find the diagonal matrix similar to $J_n$.
   (d) Find the characteristic polynomial of $J_n$.
   (e) Let $Z_n = J_n - I_n$ be the $n \times n$ matrix with zeroes on the diagonal and ones in all off-diagonal entries. Find $\det(Z_n)$, and show that $Z_n$ is invertible for $n > 1$.
   (f) Find the characteristic polynomial of $Z_n$.
   (g) Find a quadratic polynomial $f(t)$ (with coefficients depending on $n$) such that $f(Z_n) = 0$.
   (h) Use (g) to calculate the inverse of $Z_n$, expressed as a linear combination of $Z_n$ and $I_n$. (This generalizes Problem Set 7, Problem 3.)

2. Let $T : V \rightarrow V$ be a linear operator, where $V$ is finite dimensional. Suppose that $W_1, \ldots, W_k$ are $T$-invariant subspaces of $V$ such that $T_{W_i}$ is diagonalizable for each $i$. Prove that if $W_1 + \cdots + W_k = V$, then $T$ is diagonalizable.

3. Section 5.4, Exercises 13 and 20.