

**Math 110—Linear Algebra**  
**Fall 2009, Haiman**  
**Problem Set 11**

Due Monday, Nov. 16, at the beginning of lecture, along with Problem Set 10.

1. Let  $A$  be a  $n \times n$  matrix whose characteristic polynomial  $p(\lambda) = \det(A - \lambda I)$  splits as a product of linear factors.

(a) Show that  $\det(A)$  is equal to the product of the eigenvalues of  $A$ , each repeated as many times as its multiplicity as a root of  $p(\lambda)$ .

(b) Show that  $\operatorname{tr}(A)$  is equal to the sum of the eigenvalues of  $A$ , each repeated as many times as its multiplicity as a root of  $p(\lambda)$ . [Hint: consider the coefficient of  $\lambda^{n-1}$  in  $p(\lambda)$ .]

2. Section 5.2, Exercise 14(c).

3. Find formulas for  $A^n$  and  $e^{At}$ , where

$$A = \begin{pmatrix} 1 & 2 \\ 4 & 3 \end{pmatrix}.$$