Math 110—Linear Algebra
Fall 2009, Haiman
Problem Set 10

Due Monday, Nov. 16 at the beginning of lecture.

1. Let $A \in M_{n\times n}(\mathbb{Q})$ be an invertible matrix with integer entries. Prove that A^{-1} has integer entries if and only if $\det(A) = \pm 1$. Hint: For one direction of the "if and only if," use Cramer's rule—or rather, the corollary to it in Section 4.3, Exercise 25(d), which we proved in class. Deduce the other direction from basic properties of determinants.

2. Section 5.1, Exercise 3(d)

3. Section 5.1, Exercise 8

4. Section 5.1, Exercise 11

5. (a) Prove that if V is a finite-dimensional vector space over $\mathbb{C}$, $\dim(V) \neq 0$, then every linear transformation $T\colon V \to V$ has at least one eigenvector.

(b) Let $V = P(\mathbb{C})$ be the space of all polynomials over $\mathbb{C}$, and $T\colon V \to V$ the linear transformation $T(f(x)) = xf(x)$. Show that T has no eigenvector.