

Math 110—Linear Algebra
Fall 2009, Haiman
Problem Set 8

Due Monday, Oct. 26 at the beginning of lecture.

1. For each of the following statements, either prove that it is true for all systems of m linear equations in n unknowns, or give a counterexample.

(a) If the system has a unique solution, then $m \geq n$.

(b) If $m \geq n$ and the system is consistent, then the solution is unique.

(c) Given a fixed coefficient matrix A , if the system $Ax = b$ is consistent for every b , then $m \leq n$.

(d) If $m \leq n$, then the system is consistent.

2. Use Gaussian elimination to find all solutions to each of the following systems of linear equations:

(a)

$$\begin{pmatrix} -3 & -1 & 0 & 3 & -2 \\ -5 & 3 & 2 & -2 & 1 \\ -1 & 1 & 3 & 2 & 2 \\ 0 & -2 & 4 & 9 & 1 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{pmatrix} = \begin{pmatrix} 8 \\ 6 \\ -4 \\ 11 \end{pmatrix}$$

(b)

$$\begin{pmatrix} -3 & -1 & 0 & 3 & -2 \\ -5 & 3 & 2 & -2 & 1 \\ -1 & 1 & 3 & 2 & 2 \\ 0 & -2 & 4 & 9 & 1 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{pmatrix} = \begin{pmatrix} -12 \\ -11 \\ -6 \\ -13 \end{pmatrix}$$

3. Section 3.4, Exercise 5