Math 110—Linear Algebra
Fall 2009, Haiman
Problem Set 6

Due Monday, Oct. 12 at the beginning of lecture.

1. Section 2.4, Exercise 9.

2. Section 2.4, Exercise 16.

3. Prove or disprove the following statement: the set of invertible linear transformations from V to W is a subspace of $\mathcal{L}(V, W)$.

4. Let R be the rotation in $\mathbb{R}^3$ about the x-axis, by $\pi/4$ in the direction from the y-axis towards the z-axis. Let S be the rotation in $\mathbb{R}^3$ about the z-axis, by $\pi/4$ in the direction from the x-axis toward the y-axis.

(a) Find the matrices with respect to the standard basis in $\mathbb{R}^3$ of R, S and RS.

(b) Assuming that RS is also a rotation (in fact, it is true that the composite of any two rotations is a rotation), find a vector in the direction of the axis of rotation for RS. Hint: such a vector v satisfies the equation $RS(v) = v$.

5. Let A and B be the matrices of the rotations R and S in Problem 2. Find a change of coordinate matrix Q such that $B = Q^{-1}AQ$.

6. Let V be a finite dimensional vector space. Let α, β, γ and δ be ordered bases of V.

(a) If the change of coordinate matrices $[I]_\alpha^\beta$ and $[I]_\gamma^\delta$ are equal, does it follow that $\alpha = \gamma$ and $\beta = \delta$?

(b) If $[I]_\alpha^\beta = [I]_\alpha^\gamma$, does it follow that $\beta = \gamma$?

Justify your answers.