Math 110—Linear Algebra
Fall 2009, Haiman
Problem Set 2

Due Monday, Sept. 14 at the beginning of lecture.

1. Section 1.5, Exercise 3.

2. Let S be the subset

$$\{\sin^2(x),\ \sin(2x),\ \cos(2x),\ 1\}$$

of the vector space $\mathcal{F}(\mathbb{R}, \mathbb{R})$. Which subsets of S are linearly dependent and which are linearly independent?

3. Prove that if $S = \{v_1, \ldots, v_n\}$ is a finite, linearly independent set of vectors in a vector space V, then every vector $w \in \mathrm{Span}(S)$ has a unique expression as a linear combination

$$a_1v_1 + \cdots + a_nv_n.$$

4. Find a basis of the subspace of symmetric matrices in $M_{3\times 3}(\mathbb{R})$. What is the dimension of this subspace?

5. Prove that if V is a vector space over $\mathbb{F}_2$ with finite dimension n, then V is a finite set. How many elements does it have?