Matrix Computations & Scientific Computing Seminar

Organizer: James Demmel & Ming Gu

Wednesday, 11:00am–12:00pm, 380 Soda

April 06 **Artem Napov**, LBNL

An algebraic multigrid method with guaranteed convergence rate

Applications in science and engineering frequently require the solution of linear systems of equations. As the number of unknowns grows, driven, for instance, by the needs of better quality and higher accuracy of the modelling, the issues of solution time and computational resources become increasingly important. In such context, scalable iterative methods represent often an attractive (if not the only possible) approach. Among them, multigrid methods are known to exhibit optimal convergence rate in many applications; that is, to substantially (e.g., by a factor of 106) reduce the residual norm in few iterations.

In this talk, we show how to design an algebraic (or black-box) multigrid method with guaranteed optimal convergence rate. In particular, we consider multigrid methods based on aggregation and prove a bound on convergence rate for systems with symmetric diagonally dominant (M-)matrices.

The cornerstone of our approach is an upper bound on the two-grid convergence rate which is local and accurate. By local we mean that the convergence rate can be bounded above by computing separately for each aggregate a parameter which in some sense measures its quality. It is accurate since, assuming the aggregation pattern sufficiently regular, we show that the bound is asymptotically sharp for a large class of elliptic boundary value problems, including problems with variable and discontinuous coefficients.

The two-grid estimate is then used to design an automatic aggregation procedure which builds aggregates ensuring that the quality measure is above a chosen threshold. Next, we combine it with a suitable cycling strategy that, given the known bound on the two-grid convergence rate, ensures a level-independent convergence in multilevel setting. Numerical experiments demonstrating the potentialities of such approach are presented and discussed.