SECTION 1.3

    b. False. Lansing is the capital of Michigan.
    c. False. Massachusetts is a state and not the capital of Boston, a city.
    d. False. Albany is the capital of New York.

4:  a. $x$ is still $0$ since the condition is false.
    b. $x$ is still $1$ since the condition is false.
    c. $x$ is $1$ at the end since the condition is true and therefore the statement $x := 1$ is executed.

6:  a. Some student in your class has taken some computer science course.
    b. There is a student in your class who has taken every computer science course.
    c. Every student in your class has taken at least one computer science course.
    d. There is a computer science course that every student in your class has taken.
    e. Every computer science course has been taken by at least one student in your class.
    f. Every student in your class has taken every computer science course.

11: a. $\forall x L(x, Jerry)$
    b. $\forall x \exists y L(x, y)$
    c. $\exists y \forall x L(x, y)$
    d. $\forall x \exists y \neg L(x, y)$
    e. $\exists x \neg L(Lydia, x)$
    f. $\exists x \forall y \neg L(y, x)$
    g. $\exists x (\forall y L(y, x) \land \forall z ((\forall w L(w, z)) \rightarrow z = x))$
       Or, equivalently: $\exists x \forall z (\forall y L(y, z)) \leftrightarrow z = x$
    h. $\exists x \exists y (x \neq y \land L(Lynn, x) \land L(Lynn, y) \land \forall z (L(Lynn, z) \rightarrow (z = x \lor z = y)))$
    i. $\forall x L(x, x)$
    j. $\exists x (L(x, x) \land \forall z (L(x, z) \rightarrow x = z))$
       Or, equivalently: $\exists x \forall y (L(x, y) \leftrightarrow x = y)$

19: a) T b) T c) F d) F e) F f) F

39: Consider $P(x)$ to be a fixed proposition. Then the statements only depend on the truth value of the proposition $A$. We need to show that the two sides agree whether $A$ is true or false.
a. If $A$ is true, then the L.H.S. (left hand side) becomes $\forall x P(x)$ by the identity law in Table 5 in section 1.2. The R.H.S. becomes $\forall x P(x)$ also by using the identity law. So L.H.S. = R.H.S. and they must have the same truth value. If $A$ is false, then L.H.S. becomes $\mathbf{F}$ by the domination law in Table 5. R.H.S. becomes $\exists x (\mathbf{F}) = \mathbf{F}$ (assuming that our universe of discourse is not empty). So L.H.S. = R.H.S. Therefore, R.H.S. and L.H.S. have the same truth value regardless of the truth value of $A$, so they are logically equivalent.

b. Using the same procedure as above, if $A$ is true, then L.H.S. becomes $\exists x P(x)$ and R.H.S. becomes $\exists x P(x)$ as well. If $A$ is false, L.H.S. becomes $\mathbf{F}$ and R.H.S. becomes $\exists x (\mathbf{F}) = \mathbf{F}$. Therefore the two sides are logically equivalent.

\section*{SECTION 1.4}

1:  
   a. $\{-1, 1\}$
   b. $\{1, 2, 3, \ldots, 11\}$
   c. $\{0, 1, 4, 9, 16, 25, 36, 49, 64, 81\}$
   d. $\emptyset$

8: The Venn Diagram should consist of 3 circles labeled $A$, $B$ and $C$, where circle $A$ is contained in circle $B$ and circle $B$ is contained in circle $C$. There should be a box around circle $C$ labeled $U$ for universe, ie the set of discourse.

9: Let $A \subseteq B$ and $B \subseteq C$. Then for all $x$, $x \in A \rightarrow x \in B$ and $x \in B \rightarrow x \in C$, so $x \in A \rightarrow x \in C$, ie, $A \subseteq C$.

21: $\emptyset \times A = \{(x,y) | x \in \emptyset \text{ and } y \in A\} = \emptyset = \{(x,y) | x \in A \text{ and } y \in \emptyset\} = A \times \emptyset$. The second and third equalities come from the fact that $\emptyset$ has no elements, so there is no such $x \in \emptyset$ or $y \in \emptyset$. The first and fourth equalities come from the definition of Cartesian product.

\section*{Section 1.5}

1:  
   a. The set of students who live within 1 mile of school and who walk to classes.
   b. The set of students who live within 1 mile of school or who walk to classes (or who do both.)
   c. The set of students who live within 1 mile of school but do not walk to classes.
   d. The set of students who walk to classes but live more than 1 mile away from school.

7:  
   a. We can prove that $\cup$ is commutative by using the fact that $\lor$ is commutative. $A \cup B = \{x | x \in A \lor x \in B\} = \{x | x \in B \lor x \in A\} = B \cup A$. 
b. We can prove that \( \cap \) is commutative by using the fact that \( \wedge \) is commutative. \( A \cap B = \{x | x \in A \wedge x \in B\} = \{x | x \in B \wedge x \in A\} = B \cap A \).

19: a. Let \( A \cup B = A \). For all \( x \in B \), \( x \in A \cup B \), so \( x \in A \). So we have \( x \in B \rightarrow x \in A \), hence \( B \subseteq A \).

b. Let \( A \cap B = A \). For all \( x \in A \), \( x \in A \cap B \), so \( x \in B \). So we have \( x \in A \rightarrow x \in B \), hence \( A \subseteq B \).

c. Let \( A - B = A \). For all \( x \in B \), \( x \notin A - B \), so \( x \notin A \). So we have \( x \in B \rightarrow x \notin A \), hence \( A \cap B = \emptyset \).

d. \( A \cap B = B \cap A \) is true for all sets \( A \) and \( B \), (see problem 7b above!) so this tells us nothing about the sets \( A \) and \( B \).

e. Let \( A - B = B - A \). Let \( a \in A \). If \( a \notin B \) then \( a \in A - B \) but \( a \notin B - A \). But this is a contradiction since those sets are equal, so we must have \( a \in B \). Likewise, if \( b \in B \), we must have \( b \in A \). Therefore, \( A = B \).

35: a. 
\[
\bigcup_{i=1}^{n} A_i = \{1\} \cup \{1, 2\} \cup \{1, 2, 3\} \cdots \cup \{1, 2, \ldots, n\} = \{1, 2, \ldots, n\} = A_n
\]

b. 
\[
\bigcap_{i=1}^{n} A_i = \{1\} \cap \{1, 2\} \cap \{1, 2, 3\} \cdots \cap \{1, 2, \ldots, n\} = \{1\} = A_1
\]

37: a. 
\[
\bigcup_{i=1}^{n} A_i = \{0, 1\} \cup \{0, 1, 00, 01, 10, 11\} \cup \cdots \cup \{0, 1, 00, \ldots, 00, 00, 00, 01, \ldots, 11\} = A_n
\]

b. 
\[
\bigcap_{i=1}^{n} A_i = \{0, 1\} \cap \{0, 1, 00, 01, 10, 11\} \cap \cdots \cap \{0, 1, 00, \ldots, 00, 00, 00, 01, \ldots, 11\} = A_1
\]

**SECTION 1.6**

3: a. \( f(S) \) is not a function. For example, \( f(1010) = 2 \) or \( 3 \), so \( f \) does not assign exactly one integer to the bit string 1010.

b. \( f(S) \) is a function. It assigns exactly one integer to each bit string. For example, \( f() = f(0) = f(00) = 0, f(1) = f(100) = 1 \).

c. \( f(S) \) is not a function. \( f \) does not assign any integer to the string 0, for example.
8: a. \( f \) is one-to-one.
   b. \( f \) is not one-to-one since \( f(a) = f(b) = b \).
   c. \( f \) is not one-to-one since \( f(a) = f(d) = d \).

13: The following are possible answers. Other answers are in the back of the book. For each \( f \) below, \( f : \mathbb{Z} \rightarrow \mathbb{P} \) where \( \mathbb{P} \) is the set of positive integers.

   a. \( f : x \mapsto \left( (3x - .5) \right)^2 + 1 \) is one-to-one but not onto. For example,
      \[ f(0) = 1; \quad f(1) = 4 + 1 = 5; \quad f(-1) = 16 + 1 = 17; \quad f(2) = 25 + 1 = 26; \]
      \[ f(-2) = 49 + 1 = 50; \quad f(3) = 64 + 1 = 65. \]
      Another possible answer is \( f : x \mapsto 2|x| + 1 \) if \( x \) is positive and
      \( f : x \mapsto 2|x| + 4 \) if \( x \) is negative, and \( 0 \mapsto 2 \).

   b. Some possibilities for \( f \) onto but not one-to-one:
      \[ f : x \mapsto \lfloor x/10 \rfloor; \]
      \[ f : x \mapsto n \text{ where } x \text{ has } n \text{ digits (in base-10 representation)}. \]
   c. \( f \) both onto and one-to-one: \( f : x \mapsto \lfloor 2x + .5 \rfloor + 1 \). For example,
      \[ f(0) = 1; \quad f(1) = 3; \quad f(-1) = 2; \quad f(2) = 5; \quad f(-2) = 4. \]
      \[ f : x \mapsto \text{the placement of } x \text{ in the list } 0, 1, -1, 2, -2, 3, -3, \ldots. \]
   d. Some examples of \( f \) neither onto nor one-to-one:
      \[ f : x \mapsto 1; \]
      \[ f : x \mapsto \text{the first digit of } x \text{ (ignore any negative signs)} \text{ if } x \neq 0 \]
      and \( x \mapsto 1 \) if \( x = 0 \).

20: Yes. The statement is even true if we ignore the condition that \( f \) is one-to-one. Let \( f \circ g \) be one-to-one. Then \( \forall x \forall y (f(g(x)) = f(g(y)) \implies x = y) \).
      Let \( g(x) = g(y) \). \( f(g(x)) = f(g(y)) \) since \( f \) is a function, so by the above statement, \( x = y \). Thus, \( \forall x \forall y (g(x) = g(y) \implies x = y) \), i.e. \( g \) is one-to-one.

31: \( f : A \rightarrow B \).
\[
f^{-1}(S) = \{ x \in A | f(x) \notin S \} = \{ x \in A | x \notin f^{-1}(S) \} = \overline{f^{-1}(S)}
\]
54: The following graphs were generated using Maple. Unfortunately, they don't show the open circles at the end of the segments, and the part of the graph on the y axis is invisible. Also, the graph for c) should be discontinuous, not continuous as it's drawn. But at least this will help you see if you got the right shape.

> plot(floor(3*x-2), x = -.5..1.5, discont=true);

> plot(ceil(.2*x), x = -10..10, discont=true);
\begin{verbatim}
> plot(ceil(-1/x),x=-1..1,y=-10..10);

> plot(floor(x^2),x=-3..3,discont=true);
\end{verbatim}
\texttt{> plot(ceil(x/2)*floor(x/2),x=-6..6,discont=true);}

\texttt{> plot(ceil(x/2)+floor(x/2),x=-4..4,discont=true);}
> plot(floor(2*(ceil(x/2)+.5)), x=-6..6, discont=true);